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# MEASUREMENT UNITS AND MODES: THE INDIAN CONTEXT

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*Measurement is a topic area that makes strong connections between school mathematics and real life. Although the international system of units is followed in most countries and in most school science and mathematics curricula, multiple systems of units including the old British system and informal units of convenience still prevail in the everyday world of commerce in the Indian context. In this paper, we document some of the units that are still being used in the informal sector of the industry, in which many children from low-income background participate. We discuss the variety of modes of quantification in informal measurement contexts, and finally we discuss some possible implications of these findings for the school mathematics curriculum.*

*Measurement, Measurement units, Dematematization, Quantification*

## INTRODUCTION

One of the goals of inclusive mathematics education is to make strong connections between school mathematics and the life of children outside the school. When school mathematics is seen as relevant and useful and is perceived by the culture as valuable, then the motivation for and participation in learning school mathematics is enhanced. This is one of the assumptions written into the most recently adopted school curriculum framework in India (NCERT, 2005). From this perspective, measurement is an important topic area because it makes strong connections between school mathematics and real life. Measurement is ubiquitous in society, whether in relation to activities in the home or the workplace. In India, as in many developing countries, children from poor urban homes are often exposed to or participate in work related activity at home or in the neighbourhood, which contributes to the household income. In the course of such activity, they acquire knowledge that includes mathematical aspects. Often such knowledge is related to measurement, and may involve familiarity with different measurement units, ability to estimate quantities, or knowledge of the costs of different kinds of materials or goods.

In this paper, we explore measurement related aspects prevalent in the culture in an urban Indian setting through the prism of the experience of school children. Such settings may be similar to the ones which children from low income urban homes from other places in the country are exposed to. The broader purpose of this inquiry is to explore how such knowledge can help children in learning mathematics in the school. Here, however, we restrict the discussion to the different kinds of measurement units that children might encounter, and the variety of modes involved in ascribing numbers to quantities. We briefly outline the theoretical framework provided by the notion of dematematization, put forth by other researchers, which is useful to situate informal measurement knowledge. Finally, we discuss briefly what the implications of the existence of such knowledge might be for the curricular content or the pedagogy of school mathematics. Some of the measurement units used in the informal sector have been prevalent in Indian society in the past, and indeed, some have even been a part of the school mathematics curriculum. Hence it is appropriate to set a historical background to the discussion, which we provide through an exploration of a school mathematics textbook from the late $19^{th}$ – early $20^{th}$ Centuries.

## HISTORICAL BACKGROUND

In most countries around the world, the international system of units has been adopted officially and this is the only system that is included in the school mathematics and science curriculum. This was not the situation around a hundred years ago. For example, in India, multiple systems of units were used for currencies, weight, volume and time measurement up to the 19th Century. Before British rule in India, which lasted from the mid-nineteenth to the mid-twentieth Century, the major unifying force in the Indian sub-continent was the Mughal power. Under Mughal rule ($16^{th}$ to $18^{th}$ Century), a relatively uniform monetary system was adopted over much of India, which was continued and gradually modified by the British (Prakash, 2006). In British India, older Indian units of measurement and their local variations co-existed with the units introduced by the British, a situation that is reflected in the textbooks of the late $19^{th}$ Century.

The textbook that we use to illustrate the historical background is a mathematics textbook meant for use in the primary school in the Western Indian state of Maharashtra written by Gopal Krishna Gokhale (1866-1915). Gokhale was a major Indian nationalist leader, who taught mathematics in a reputed college. The textbook was written and revised in the last two decades of the $19^{th}$ Century, although the edition that we cite is one that was in print later (Gokhale, 1921). One of the striking features of these textbooks is the proportion of the curriculum dealing with measurement. The chapters on measurement do not include a discussion of the concepts of measurement, the act of measurement or measuring instruments, but deal rather with the kinds of numbers and units that are used in measurement. Detailed information about the kinds of units and their inter-conversion was provided in the textbooks and students were presumably expected to use them while solving problems.

Gokhale's textbook begins by distinguishing two kinds of numbers, those that indicate pure value and those that indicate quantity. Examples of the first kind are "two", "five" and "hundred", i.e., numbers without any object or unit names attached. Examples of numbers that indicate quantity are "ten men", "twenty cows" and "thirty mangoes". Here the units, respectively "1 man", "1 cow" and "1 mango" are natural or "self-evident" units. On the other hand, when we say "three cubits", the unit "cubit" is conventional (a unit of length roughly equal to half a yard). Hence quantities may be indicated by units fixed naturally or by convention.

Conventional units to measure quantities are introduced in the textbook only after a substantive treatment of the four arithmetic operations with numbers. Four kinds of quantities or measures are explicitly distinguished, those that specify money, weight, magnitude (length, area and volume) and time. The author goes on to say that it would have been convenient to have decimal sub-units; currency units could for example be related as 10 pies = 1 anna and 10 annas = 1 rupee. However, decimal relations are not the ones found in the real world where 12 pies = 1 anna and 16 annas = 1 rupee. Hence students need to learn how to interconvert between sub-units and larger units, to notate measures in mixed units and to learn the arithmetic associated with compound measures notated in this manner.

The textbook contains several chapters devoted to exercises on the addition, subtraction, multiplication and division of compound measures. For example, a sum of money of 8 rupees, 3 pavalis, 0 annas and 7 pies would be notated as “8 3 0 7”, where the inter-conversions are 1 rupee = 4 paavalis = 4 × 4 annas = 4 × 4 × 12 pies. Similar notation for compound measures for weight, volume and time were also used. The “carry over” procedures for adding compound measures is more complicated than simple addition: 12 pies need to be regrouped as 1 anna, and 4 annas need to be regrouped as 1 paavali, and so on. The ability to compute with compound measures is, as indicated by the length of chapters dealing with this topic, perhaps the most important goal of primary mathematics.

The textbook reveals something of the complexity of weights and measures prevalent in the world of commerce of the period. A chapter in the textbook is devoted purely to information about measurement units of various kinds, presented in the form of tables. For example, four different systems of weight measures for weighing metals such as gold and silver are presented: the system in the state of Maharashtra, the “old” system, the system prevalent in the city of Bombay and the system in England. Each system has its own units and sub-units. The same attribute of weight was measured using different units for different objects. Units used to measure grain by weight were different from the units used to measure precious metals like silver and gold and the units used to measure quantities of salt and sand. It is not surprising that the tables for these units run into a few pages. The chapter included such details as the slightly different values of the standard unit of currency, namely the 'rupee', in different cities.

The extensive treatment of arithmetic and the detailed exercises with a variety of units suggest that such skills were needed and valued in the everyday world of commerce around the time when the textbooks were written. A few decades later, we find a textbook having the title “New Arithmetic”. In this textbook from the 1930s, the detailed tables of units have disappeared, compound operations are completely omitted and the curriculum begins to take a recognizably modern shape (Deshmukh, 1935). What is striking in the light of recent debate in mathematics education, is that the textbooks from a hundred years ago show a strong connection with life outside school, while educators worry about the lack of such connections in modern textbooks.

## STANDARDIZATION AND DEMATHEMATIZATION

Up to the first half of the 20th Century, the British government permitted the use of the older system of units alongside the British Imperial system. However, with the increasing adoption of the British units, the school mathematics textbooks no longer aimed to teach the older systems, and as we have seen, no longer include “compound operations”. We might say that these mathematical skills became redundant. In the 1960s, the government of independent

India adopted the metric system of units and gradually units in the British system and their interconversion with metric units were omitted from school and college textbooks. One may see this as part of a trend of "demathematization", a notion that has been discussed by several mathematics education researchers. "[Demathematization] also refers to the trivialisation and devaluation which accompany the development of materialized mathematics; mathematical skills and knowledge acquired in schools and which in former time served as a prerequisite of vocation and daily life lose their importance." (Keitel, Kotzmann & Skovsmose, 1993, quoted in Jablonka & Gellert, 2007, p. 8) Dematheratization with respect to explicit knowledge and skill accompanies the process of the mathematization of society, i.e., the incorporation of implicit mathematical knowledge in artefacts, instruments and practices. "The greatest achievement of mathematics... can paradoxically be seen in the never-ending, two-fold process of (explicit) *demathematizing* of social *practices* and (implicit) *mathematizing* of socially produced objects and techniques." (Chevellard, 2007, p. 60, emphasis original)

The arithmetic of the compound operations was needed because decimal numbers were used to compute with systems of units that were not decimal. The skills of computing with a variety of compound units became redundant upon the adoption of a standardized system of units and measures at the national and the international level. Standardization is one of the means by which demathematization takes place. Other ways are the incorporation of arithmetic in artefacts and devices: calculators make paper-pencil calculation redundant; comparative EMI tables make it unnecessary to calculate interests. Demathematization is also devaluation and hence impacts learning opportunities which are framed by what the culture values and perceives as useful.

While the notion of demathematization of explicit mathematical skills and knowledge outlines a broad trend, there are also counter-trends to this overall process. The older units prevalent in India have not entirely disappeared from the world of commerce and industry. In domains that have cultural or religious significance, older units are still used. A familiar example is the buying and selling of gold in India, which is common before important occasions like a wedding. People are familiar with the traditional units in which the weight of gold is measured and these are still being used. In Northern India, the common traditional unit is the tola (about 11.66 gram), while in South India the common unit is the pavan (8 grams). There is a trend of increasingly measuring gold in "grams" as gold becomes a commodity, and "hallmark" certification of purity gains acceptance. Another context in which older units prevail is traditional architecture. Traditional architects, called "*sthapatis*" are needed to build temples, and sometimes even homes. The basic dimension of a temple or home has numerological and religious significance, and hence is expressed in the traditional unit of "*hastas*" or "cubits" (1 cubit is taken to be equal to 2' 9", which is somewhat longer than a cubit in other contexts, Ganapati Sthapati, 2008, p. 792). Even in the construction of homes, it is more common to use feet to indicate the dimensions of a plot of land, and square feet to indicate the areas of homes. Most builders work with measurements in feet and inches, even though drawings may be in meters and millimeters.

## MEASUREMENT UNITS IN HOUSEHOLD BASED ECONOMY

Another counter-trend to demathematization is the circulation of mathematical knowledge in the informal economy. Many researchers have pointed to the fact that children who participate in the informal economy possess mathematical abilities and skills, which are different from those that they learn in school. Some researchers have explored the nature of such knowledge and its differences from school mathematics (Resnick, 1989), while others have explored implications of such knowledge for teaching school mathematics (Nasir, Hand & Taylor, 2008). The framework provided by the concepts of mathematization-demathematization as twin processes that affect the circulation of mathematical knowledge in society, situates out-of-school mathematical knowledge of children in larger processes of change. The general trend of a shift from craft based industry to large scale factory based manufacture leads to deskilling and to the expert craft based knowledge becoming redundant. Demathematization is a part of this process. The emergence of the informal sector is a counter-trend to industrial factory-based production, and hence creates its own need for mathematical knowledge. In the informal sector therefore, we see not only the circulation of a form of arithmetic knowledge and skill, but also knowledge related to measurement that may not be “standardized”. One aspect of such knowledge is the familiarity with and use of informal measurement units. Another aspect is the experience of “non-standard” modes of quantification. We shall discuss these below.

Children from low-income families who participate in the informal sector of the economy are often exposed to a variety of “non-standard” or older units that are still being used. Studies done elsewhere of 'everyday mathematics' have shown that children's knowledge of a variety of units used in 'everyday' activities are mostly based on convenience and are supported by the prevalent practices in the work-domain or in daily usage. To our knowledge, only a few studies have looked into different units emerging from 'everyday mathematical' practices and have characterised the units, exceptions being Millroy's study with carpenters (1992) or Nunes and Bryant's study (1996).

### Description of the study

The sample for this study was drawn from Grade 6 students of an English medium and an Urdu medium school run by the Municipal Corporation of Greater Mumbai, located in a large low-income area in Mumbai, India. Many of the children in this area were involved in a variety of ways in house-hold based economic activity. The researcher (i.e., the second author) began by observing the classrooms followed by informal discussions with the students to get a broad picture of the nature of their daily activities that have aspects of mathematics and the nature and extent of their everyday mathematical knowledge. This helped in getting an initial understanding of the variation in children's out-of-school mathematical knowledge as well as their involvement in the economic activity. Two rounds of interviews were conducted after the classroom observations and initial discussions: a semi-structured interview to understand child's family-background, family's socio-economic status, parents' occupations, productive work done at home/elsewhere and child's involvement in them, and an interview based on a structured questionnaire to understand students' basic arithmetical knowledge. The data used for this paper is drawn from all phases of the study including informal visits to the house-holds and small manufacturing units and discussions held with adults in these locations.

**Measuring Units**

Children in the study often participate in and are exposed to a variety of activities that characterize the house-hold based informal economy such as tailoring, sequin-stitching, leather work, catering, etc. A variety of formal and informal units are used in such work. By formal units, we indicate units belonging to the International system and also those that have remained from an older system of units. Informal units are units of convenience, and may not be defined precisely quantitatively. We describe some of the informal and formal units in use below.

**Units used in Zari work**

Zari work essentially involves thread work and stitching a variety of decorative sequins on to pieces of cloth, and is a common occupation in the low-income area where the study is being conducted. Workers and many students (some school students do part-time zari work) are familiar with different units in which sequins and other raw-materials are sold in the market and required for certain specific tasks. Some decorative elements are sold by weight in pound units and some in gram and kilogram, some are sold in 'laris' (a bunch of tiny spherical sequins strung on a thread), while some items are sold by metre. 'Pound' is a unit from the older British system that is no longer a part of the school curriculum.

Some decorative elements are tiny, light in weight, but expensive. These are frequently measured using the informal unit of a 'mutthi' or 'fistful'. Apprentices make an estimation of the amount of some of the raw materials required in fistfuls.

**Units used in leather work**

Leather work includes the making of bags, wallets, purses, files, shoes, etc. Both length and area units are used here. Standard units from the British system like inch, foot, and international standard units like metre are both used. Some indigenous units are also used, viz. 'waar', 'desi', etc. 'Desi' refers to a square piece of leather of dimension 4 inches × 4 inches. Typically, a leather piece of dimension 'one foot by one foot' is used for bag-making which is covered by exactly 9 desi. 'Waar' (also called 'gaj' – the Hindi word for yard) is a length unit used for measuring zips. The sizes of zips are indicated using the counting numbers (No. 1 to 8) that show varying width and size.

**Units used in stitching work**

'Gross' (usually pronounced as 'grus') is the unit used to count decorative 'stones' which are fixed in earrings or stitched on clothes. This is again a unit from the British system: 1 gross = 12 dozen = 144 pieces. However, for the convenience in calculation, 1 gross is often taken to be 140 pieces.

**Other units**

Children are also familiar with the traditional measurement units for pieces of land, viz. 'kattha', 'bigha', (1 'bigha' = 20 'kattha', 1 kattha is about 1300 square feet, but varies from place to place). The British system of 'acres' and 'square feet and metric units like square metres are also used. 'Square feet' is the common unit for measuring the floor-size of apartments in the cities and prices are quoted using this unit.

Flower garlands are commonly used in India for ornament and for religious occasions. These are typically sold in the market by cubits measured from the elbow to the tip of the finger. An informal unit often encountered in the everyday world in a city like Mumbai is 'cutting', which is used to refer to half a cup of tea in most tea-joints in and around the location of the study. 'Cutting'-tea is a popular beverage in most roadside eateries and served throughout the day, and in practice can be more than half a cup and is usually served in a glass tumbler. Other units like 'two by three' or 'three by four' that indicate of two cups of tea divided among 3 persons or 3 cups among 4 persons – are also commonly used.

### Measuring instruments

In small manufacturing, length is often measured using a convenient template length unit that is usually non-standard. In tailoring or leather work, measuring tapes and scales (commonly 24-inch steel scale and 60-inch plastic tape) are used to cut 'farma' (templates) which are then used to indicate measurement and design of different parts of the object being manufactured. For example, 'farma' of shirt-collars and shirt-pockets, 'farma' of wallets and purses are commonly used in garment manufacturing units and leather workshops respectively. The 'farma' for making wallets is often cut from a canvas which come in the length of 33" x 39" (1 metre = 39 inches approx.). In the traditional construction of wooden fishing boats on the Eastern coast of India, length is usually measured in terms of a template unit using a rope which has a knot tied at each of its two ends. Smaller lengths are measured by folding the rope one or more times (Mukhopadhyay, 2011).

## MODES OF QUANTIFICATION

The manner in which measurement occurs in everyday commerce or in work contexts, especially in the informal economy may be significantly different from the picture of measurement in scientific or engineering contexts characterised by precision and exact quantification. We see a diversity of modes of quantification in the informal context.

- In some situations, quantification is convenient, imprecise and limited. The example of a "cutting" tea fits this description. Cutting refers to a half filled glass tumbler or "cup" (itself a unit of convenience), which may actually be a little more than half a cup. It is sold at half the price of a full cup, sometimes a little more. The "cutting" functions as a separate stand alone unit, but not as a part of a fraction of a cup. Nor are cuttings partitioned further. Vendors usually do not sell tea in other fractional quantities, but may do so for a group of favoured vendors, creating fractions like ¾ or even 5/7, although such fractions of a cup are indeed made while serving tea in other cities in India.
- Some units like the "mutthi" or "fistful" are imprecise, but can be partitioned: shops may be willing to sell half a mutthi of tea powder or poppy seeds. Indeed, in the context of cooking, a handful is a unit that may be partitioned or reduced

proportionately when the number of servings is changed. Like the hand span, this is a commonly used and "handy" body based unit.

- In the context of tailoring, length is measured using the "inch tape", an inexpensive plastic tape that has both inches and centimetres marked on it. Students often confuse between these units from two different systems and are not clear about the distinction. Despite this, they may be able to carry out measurements of acceptable accuracy by reading off the length from the tape. Such measurement is fully quantified, but the quantification is opaque, and the measurement itself is critically dependent on the integrity of the artefact. Children may for example be unable to use a broken scale or tape, and may make errors if the tape is used in a non-standard way (for e.g., measuring from a point other than zero).
- An artefact like the measuring tape may become so familiar that children may estimate lengths with surprising accuracy without the use of the tape. Similarly they may accurately estimate the weight of an object that is within the range of a few kilograms. Here we see an example of how an artefact in the form of a measuring tool is integrated with bodily proprioception to create a form of embodied knowledge or skill. However this does not necessarily imply that children are aware of how weight or length is quantified.
- We also find instances of precise quantification using a non-standard template length, as in the instance of the building of traditional fishing boats referred to earlier. The master builder fixes a basic length which is specified by a length of rope between two knots. All measurement is done with this rope, which may be folded several times to measure small lengths precisely. Although the measuring rope is itself unmarked and bare, the quantification is more transparent in the use of the rope than in the use of a marked tape. The rope is iterated to measure longer lengths, and folded to form fractional units that in turn function as units that can be iterated. Here too, familiarity with a particular unit may result in an embodied skill at estimating lengths without the aid of the measuring rope.
- Children exposed to tailoring or attending to customers in a garment shop are familiar with shirt sizes. Some shirt sizes are marked with a letter (for example, 'S' for small), but more commonly, a number like '38' or '40' is indicated in the label. Although most adults and many children are familiar with these shirt sizes, whether and how these numbers are obtained through measurement is not clear to most people. Children in our study interpreted these numbers as unrelated to any units like inch or centimetre, and as merely indicating increasing sizes. Only some tailors were aware that this indicates the person's chest measurement (not the shirt chest measurement) in inches. Here we have an instance of a measure familiar from experience, but whose origin in quantification is obscure.

## CONCLUSION

A study of measurement in informal work contexts reveals a situation characterized by a rich diversity of measurement units and modes of quantification. Besides informal units and units of convenience, older units may still be in use in the culture together with standard international units. Mathematics textbooks from a century ago reflect something of the diversity and richness of measurement units in the everyday world, but stress the arithmetic of conversion and computation rather than the concepts of measurement. In modern Indian school mathematics textbooks this diversity is not found and only standard units are taught using standard measuring instruments like a ruler or weighing balance. School curriculum designers do not consider it worthwhile to deal with a variety of units even though they may still be used. The framework of demathematization helps explain why informal practices and contexts have disappeared partly from social practices and wholly from the curriculum and how their importance and value is diminished. However, in household based occupations, measurement in a diversity of modes and with a variety of units always plays a role. The emergence and survival of such informal mathematics can be seen as a counter-trend to the broad process of demathematization.

When school mathematics textbooks adopt a restricted view of measurement, children may fail to see any connection between their classroom experience and the rich world of measurement outside school. Further, how an attribute is quantified may not be clear from classroom learning. Children also need to appreciate the fact that measurement as it occurs in the world of commerce or in work contexts in the informal sector may show characteristics quite different from precise, scientific measurement. The extent of precise quantification may be limited, and may be just sufficient for the purposes at hand. The quantification may be incomplete or if embedded in cultural artefacts, may be opaque. Even so, it may be embodied in the form of a skill at estimating quantities.

The prevalence of diversity in measurement units and modes in the culture suggests that more than teaching measurement as a skill, it is the conceptual aspects of measurement that are important to learn. Understanding how quantification is achieved in various modes may allow children to understand and make connections among the diverse ways of measuring that they encounter. It may lead them to appreciate the possibilities and limits of different kinds of informal measurement, and the distinctiveness of these from scientific measurement. Further, an inquiry into the history of older units still in use may provide interesting avenues of exploration and possibilities of connection with other curricular subjects. An inquiry into familiar measurement tools which embody measurement ideas in a “materialized” form, but where the process of quantification is obscure (like the inch tape or shirt sizes) can potentially become an important part of school learning. These ideas need to be explored further. However, it seems likely that more research about how measurement plays a role in the everyday world in diverse ways is likely to have an impact on the school mathematics curriculum.

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