Innovative Practices in Mathematics Education: An Overview

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Introduction

The teaching and learning of mathematics is a complex activity and many factors determine the success of this activity. The nature and quality of instructional material, the preparation and pedagogic skills of the teacher, the learning environment, the motivation of the students are all important and must be kept in view in any effort to ensure quality in mathematics education. Often when one refers to innovations, one only has in mind the first of these, namely, instructional material; even here 'innovation' commonly means teaching aids or manipulatives. A large number of such devices that are excellent aids to mathematics learning are indeed available. However, it is important to remember that the other aspects mentioned are equally important and together determine the the range of actual and possible innovations in mathematics education.

In this paper I shall discuss innovations and innovative practices in mathematics education under two broad headings. Under *pedagogic resources*, I shall include teaching aids, games, activities, models and curricula – resources that a teacher may draw upon to advance her pupils' learning. The second broad category of innovative practices belong the domain of *teacher professional development*. I shall argue th at a greater effort is needed in this domain. In the course of the discussion on pedagogic resources, I shall refer briefly to specific innovations that have been contributed by teachers, researchers and educationists. These will necessarily be only a sample. I shall also attempt to convey some of the assumptions that seem to me to guide the practice and reflections of innovators and researchers. Through these discussions, I hope to provide an overview of what is available, and to focus attention on the areas of innovation where more energy and effort must be directed.

To begin with, it is useful to briefly review the main learning outcomes that we wish children to acquire in the domain of mathematics in elementary school. The following short list will serve to remind us that the objectives are broad and cover different kinds of competencies:

- Rapid and accurate recall/ construction of the number sequence and of simple number facts related to the basic arithmetic operations
- Correct interpretation of the mathematical structure in a problem solving situation and the operations required to solve the problem
- Bug-free knowledge of procedures used for computation
- Ability to estimate numbers and quantities in various situations
- Correct understanding of complex symbols such as fractions (rational numbers) and algebraic notation
- Ability to reason mathematically; capacity to use and reason with mathematical language and symbols
- Spatial reasoning abilities including the use of numbers in geometric problems

Some of these competencies, such as estimation, reasoning and problem solving, do not receive sufficient emphasis in traditional curricula, but are important from the point of view of building life skills, and as preparation for further learning. Innovators need to address the whole range of competencies in their efforts to improve existing practices. One would expect therefore that innovative teaching practices and aids would be of different kinds depending on what competency they seek to develop or strengthen. For instance, some may be aimed at elucidating a concept, while some may aim to provide practice of a particular skill. Some innovations would be intended to develop reasoning abilities, some at making connections, and so on. I shall mention examples of these different kinds of innovations in the paper.

The cycle of innovation begins with the identification of a problem such as a difficult concept, the lack of a particular skill, a commonly found misconception or a systematic error. These reveal themselves most often in the course of classroom teaching or during assessment. It is no wonder therefore that teachers are at the frontline of innovation, a fact that deserves more attention than it receives. The teacher or innovator initiates an innovative practice, which may take the form of a teaching aid, a story, an activity or exercise, and so on. The practice goes through a period of trial where it is tested, improved and refined. The teacher herself grows in the process, in her understanding of the original problem or difficulty and in her skill in effecting the innovative practice. When a stable equilibrium is reached, the innovative practice is ready for replication and propagation among other teachers. Unfortunately, in our school system there are rarely opportunities for teachers to learn from each other. One of the most important changes required in the system, if innovative practices are to spread, improve and take root, is better peer interaction among teachers. The cycle of problem identification, initiating a solution, testing and refining it and eventual dispersion, holds for micro-innovations in the classroom as well as for broader, systemic innovations such as whole curricula, school management practices, use of new learning technologies and so on. The point to note is that innovations need to leave the innovator’s home and lead a wider life of their own. One of the responsibilities of mathematics educators is to put in place mechanisms which allow the sharing of innovative practices and to develop a discourse that can build on existing innovations and foster the development of new ones.

There are two ways in which one may structure the discussion of specific innovations. One is to focus on specific concepts in a topic or an area of elementary mathematics and discuss the innovative practices that facilitate their acquisition. The second approach could be to classify different kinds of innovative practices. In the following section I take the first approach and discuss the area of numbers and operations and some innovative practices that contribute to enhanced learning in this area. The rest of the paper follows a classificatory approach.

Numbers and Operations

One of the first steps the child must take is the mastery of the counting number sequence. She must learn the first few numbers, learn how to count further using the patterns of tens and hundreds, learn the written numerals and understand the positional values of the digits. The mastery of the counting system and written numerals takes place over a span of several years. One must remember that although the decimal place value system of representing numbers is extremely efficient, it is by no means ‘natural’. It developed historically only over a long period and only in certain cultures. We want our children to master this system, at least for numbers up to thousand, so well that the associations between the number word, the numeral and the magnitude of the number becomes automatic, unconscious and rapid. This is the foundation for all that comes later and is indeed the first pedagogical challenge for teachers, and one must add, for many parents.

The practice of chorus chanting is a response to this pedagogical challenge, and might well have been an innovative practice in the distant past. But by itself, it is insufficient if children are to learn more than the first few numbers. Even to master the number sequence up to 100, children need to understand and use the pattern of the decades. This poses hurdles since the pattern of number words shows idiosyncrasies in many languages. The South Indian languages have a more regular structure than English, which in turn is more regular than the North Indian languages. The reversed order of the digits in the two digit number words in the North Indian languages leads many children to incorrectly reverse the order while writing the numeral. Although eventually nearly all children will correct this mistake, teaching practices that sort this out earlier are valuable since they prepare students better to absorb other concepts. A common practice among teachers is to write sets of ten numbers under a drawing of the 'decade house' to which they belong: the numbers from 20 to 29 for example, belong to the 'house of twenty'. This commonly found pedagogical device is practical as well as useful. Other simple practices that can be implemented in any classroom include different kinds of oral counting. When children begin to verbalize the number sequence correctly, one can enhance their recall abilities through various 'jump-counting' exercises (counting in twos, threes, fives, tens, etc.) and corresponding written exercises. These teaching practices are used by many teachers, but neverthless count as innovations since they deal with the important pedagogical problem of having children become fluent at counting.

It is not enough if children learn the number sequence, they must also have a sense of the magnitude of the number. This requires the understanding of place value. There are a range of concrete manipulatives that are useful in representing the concept of place value. These include beads, stones, seeds, sticks, strips, matchsticks, Dienes blocks, unifix cubes, etc. They vary in terms of cost, availability, ease of use and other advantages, and teachers may want to choose whatever is most appropriate to their situation and style of teaching. Many educators have stressed the importance of actually handling and using such concrete manipulatives. They not only help build a strong concept of place value in number representation, but also help in clarifying the procedures used for operations on numbers. Let us take the example of a simple concrete representation of place value: matchstick bundles. A single matchstick stands for a unit or a one, a bundle of ten sticks stands for a ten, and a bundle of 10 tens stands for a hundred. Figure 1 indicates how these could be used to show a number as well as to show the operation of addition and subtraction.

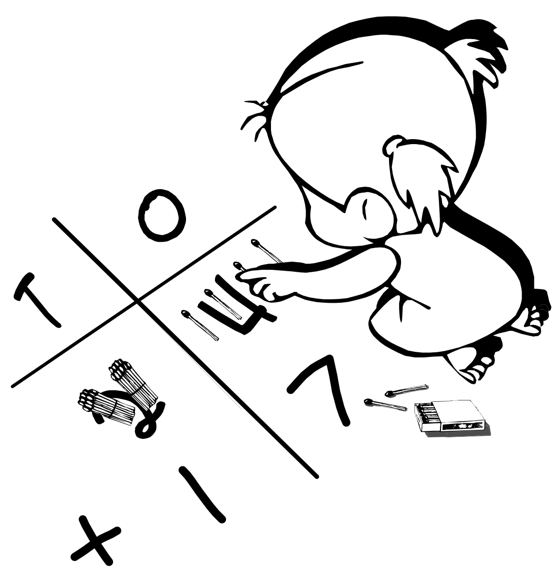

Figure 1: Using matchstick bundles to illustrate addition

When we demonstrate addition and subtraction using matchsticks with teachers, there is often a sense of discovery, a feeling that they have now understood the basis for the procedures of 'carryover' and 'borrow' that they have been teaching all along. Thus such concrete representations are useful in clarifying these concepts even for teachers.

The matchstick bundles could be used in concrete or in pictorial form to illustrate how place value is used in multiplication. Figure 2 illustrates how this could be done. Notice that icons have been used for bundles of ten, hundred and thousand. Also notice the convention used to show a `carryover'.

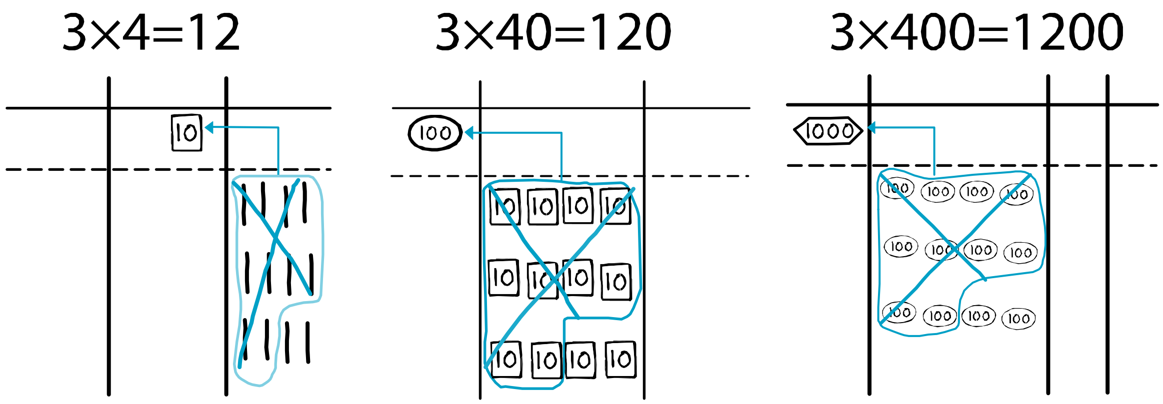

Figure 2: Illustrating patterns in multiplication

It is impractical to continue using concrete manipulatives to represent the higher place values. It may even be counter-productive, since children need to move to a more abstract and symbolic understanding. In bridging the passage from the concrete to the abstract, the stick abacus is a useful device since it combines elements of abstraction besides being a concrete model. It can also be used in pictorial form, in exercises aimed at strengthening the understanding of place value and of periods. Similarly, play money in the form of notes, coins or cheques are semi-concrete representations and are useful both as manipulatives and in pictorial form.

The foregoing discussion shows us, firstly, that many innovative practices are at the micro-level; they are everyday teaching innovations that occur in the classroom. That does not mean that they are unimportant, for they may be very effective. Secondly, there are a large number of innovative, thoughtfully produced aids that are also available. However, their actual effectiveness is dependant on the skill in using them. This is an important point and I will discuss this later under the heading of teacher professional development.

Pedagogical resources: a classification

There are a large number and variety of pedagogical resources useful in mathematics education. The following sub-sections present a classification of such resources. The classification is intended to provide a framework for the discussion of the contributions each kind of resource makes to the teaching-learning process.

Structured manipulatives

Structured manipulatives are among the most important kinds of aids. The manipulatives that were mentioned as aids to understand place value belong to this group. The structure of these manipulatives mirrors the structure of the mathematical concept: a bundle of ten matchsticks stand for a ten, opening a bundle is the process of decomposing, making a bundle is recomposing, and so on. Some manipulatives which represent place value such as Dienes' blocks have a fixed design where decomposing a place value unit into lower place value units is not possible. Here the idea of 'exchange' is used instead of 'decomposing' or `recomposing': one exchanges ten units for a ten, or 10 tens for a hundred, and so on.

Many creatively designed aids have been developed that embody some aspect of mathematical structure. Coloured blocks that can be joined together can be used to illustrate simple addition facts. Multiplication can be represented by means of arrays of objects – cubes, tiles, pegs, etc. Such arrays can be used to communicate a variety of concepts related to multiplication. The *mathemat* developed by Vivek Monteiro of Navnirmiti is a device that combines many of these possibilities. It can be used to show different display forms including arrays and linear addition facts. The kits developed by P.K. Srinivasan, one of the pioneers innovators in mathematics education, also contain well-designed, low-cost structured manipulatives for various mathematical concepts.

The topic of fractions is a notoriously difficult topic and many manipulatives have been developed that embody aspects of the structure of fractions. The most common are area models which use standard shapes: rectangles, squares and circles. The fraction chart which shows equal length strips, each divided into a different number of equal parts, placed one below the other is a linear model. An Australian group at the University of Melbourne reports success with the use of linear pipe models to illustrate place values in decimal fractions. Linear models are important since they can be connected with the positions of fractions on the number-line. Some volume models have also been developed, such as the 'fraction jars', developed at HBCSE. But these are useful only with children who are in class 5 or older, since the concept of the conservation of volume develops only slowly.

Structured manipulatives provide a base of concrete experiences on which abstract conceptions can be built. Such experiences are useful and providing for them undeniably enhances the quality of the learning experience. One must however be aware that there are limitations to what can be done with manipulatives alone. Firstly, the development of the concept at the symbolic or abstract level does not follow automatically from handling the concrete manipulatives. Much pedagogical work and creativity may be needed to let children make a transition from the concrete to the abstract level. Secondly, work also needs to be done at the abstract or symbolic level to secure the concept fully. In fact, one often sees that adults who have mastered the procedure, even if they have learnt it mechanically, are better placed to appreciate the similarity between the symbolic procedure and the concrete operation carried out with the manipulatives. The relation between experience at the concrete level and the formation of the concept is, from the cognitive point of view, a complex one and not very well understood.

Further, it is important to ensure that students indeed move to the abstract level because the power of mathematics derives from abstractions. The shift from the concrete to the abstract can be quite radical in mathematics. Evidence for this is found in the fact that what was initially abstract becomes concrete later on. The best example of this are the natural numbers. This is a purely abstract notion and takes considerable time for young children to master. However, for older children numbers function like concrete objects in their own right paving the way for more abstract concepts such as factors, multiples, prime factorization, and the concepts of algebra.

The shift from concrete to abstract is radical in another sense. While the concepts at the abstract level derive their power from their generality, this may entail a loss of easy interpretation of the concept at the lowest concrete level. Take the example of the subtraction operation. After the introduction of signed numbers, it is more appropriate to think of subtraction as the addition of the additive inverse rather than as a separate operation. This may be easy to interpret in a context involving profit and loss. But in other contexts that involve the subtraction operation, there is no easy interpretation of negative numbers. Water evaporating from a dish, or even eating a few bananas from a bunch are easily modeled by subtraction, but it is difficult to think of these as adding the additive inverse. A similar situation obtains when the rational numbers are introduced and division is absorbed into multiplication as multiplication by the reciprocal. Think of the problem 1¾ ÷ ½. This can be written as 1¾ × 2. Attempting to illustrate this equivalence in concrete terms may have the result of making a student's understanding foggier rather than clearer.

Thus concrete manipulatives must function not like a crutch but like the Wittgenstinean ladder that must be thrown away once the climbing is done. A balance needs to be maintained between the two essential components of good mathematical pedagogy: concrete scaffolding and abstract reasoning.

Mnemonic and attentional aids

Attention is a pre-condition for learning and teaching involves continuous effort at directing students' attention. So it is not surprising that devices such as flashcards that help to attract and focus attention are helpful. Other examples include number charts, multiplication charts, large pictures of notes and coins, etc. All of these are extremely useful in the classroom when the teacher is explaining or posing a problem. The blackboard is perhaps the prime example among attentional aids. School education, at least in the present era, is unthinkable without a blackboard, and nothing is more depressing than to find schools with poor blackboards. It can safely be stated that the size of the blackboard is directly proportional to its impact. I have seen schools run by enlightened NGOs where much of the wall is converted into a blackboard or a board of some sort. It is a practice in countries like Japan to use roll-up or small blackboards that can be hung on a nail in addition to the main blackboard. These are used to highlight some especially important material: a problem, a principle, a property, etc.

One may add some structure to the display device to make it more useful. For example, large roll-up blackboards can be painted with white paint to show a grid of dots or lines. At HBCSE, we call these 'dot boards'. This simple modification makes the dotboard a versatile device which could be put to the following uses:

- Illustrating the concepts of length, area and perimeter of different geometrical shapes. The dotboard is a convenient substitute for the geoboard for many purposes.
- Showing various geometrical shapes in different orientations. The dotboard makes it easy for the teacher to draw the shapes.
- Explaing concepts such as sides, vertices, etc. of plane figures .
- Illustrating reflection symmetry on a line
- Showing fractions and decimals and explaining associated concepts

A number slide is another attentional aid that is easily made. For the paper version of this aid, in a long broad strip of paper, slits are made so that another narrow strip passes through the slits and can slide

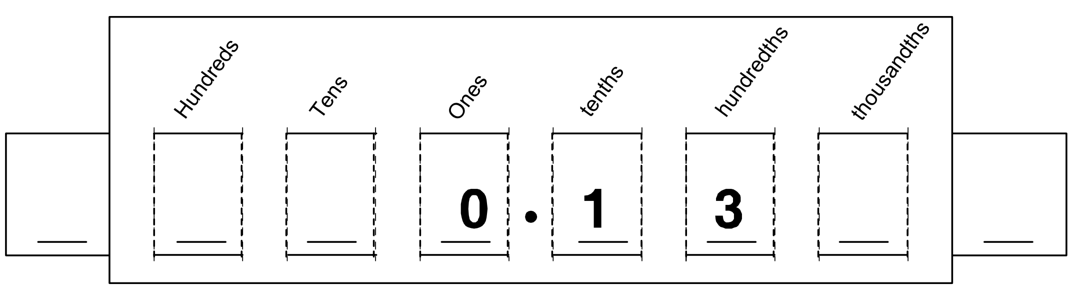

Figure 3: The number slide ***(Source: Decimal Project Website, University of Melbourne)***

back and forth (see Figure 3). The sliding strip is much longer than the broad strip and has blanks drawn which are aligned to the windows on the broad strip. On the number slide, the place values are written above each window where a digit appears. In the figure, the number slide is used to explain the recomposition of decimal fractions: 13 hundredths is 1 tenth and 3 hundredths. The slide also allows the illustration of what happens when multiplying or dividing by powers of 10 – shifting to the left or to the right. The number slide can be modified to explain place value conversion for whole numbers, and conversion between decimal units and subunits of measurement (for example, meter → decimeter → centimeter → millimeter).

Games and puzzles

Mathematics, because of the beauty of its intrinsic structure and the intellectual stimulation that it provides, can be an inherently pleasurable activity. However, many students experience it as a rather unpleasant subject. The reason for this may partly lie in the pedantic approach to the subject taken by many teachers, and partly in the fact that some mathematical skills need a lot of practice before they are internalized and easily recalled. Games can be a means of making the practice of skills less tedious, sometimes even pleasurable. The simple game of *target 100* is a good example. Two individuals or two groups may play this game. Each group takes turns to add a single digit number to a total that keeps growing. The first group that makes the total equal to 100 wins.

An activity that makes addition practice interesting is the *palindrome chart*. In this activity, on the chart of two digit numbers, children colour the numbers which have the same palindromic order with the same colour. The *palindrome chart* illustrates an important general principle: exercises that are embedded in activities that are meaningful and interesting are preferable to exercises that involve just repeated practice with a certain skill. Palindromic numbers are interesting in themselves – they are the numbers which read the same even if they are read backwards, for example, 2442. If a number is not palindromic, then one adds to it the reversed number. For example 34 is not palindromic, so we add its 'reverse', 43 to it, obtaining 77, which is palindromic. Since we obtained a palindromic number with just one step of reversing and adding, 34 has palindromic order 1. Other numbers may need two or more steps of reversing and adding before we obtain a palindromic number, and hence have higher palindromic order.

Children notice the pattern for 'one-step' palindromic numbers. These are the numbers where the sum of the digits is less than 10. They begin to look then for other patterns. So the addition that they do is embedded in this interesting activity of finding patterns. Moreover, the pattern is critically dependant on the carryover procedure, since carryovers 'spoil' the chances of obtaining a palindromic number. This is precisely where we want children to focus attention since children need to master the 'carryover' procedure in vertical addition. These factors make the palindrome chart especially appropriate and successful. In general, children are interested in patterns and seek them out; this is a part of their general drive to acquire knowledge about the world around them.

Many interesting games and puzzles can be designed by associating numbers with traditional game devices such as boards, cards and dice. Dominoes are a well-known example. Here are a few more.

- *Jod-ghata ka khel* to found in the Eklavya books. In one version of this game, children roll two dice marked with appropriate numbers. They may do any one of the four operations with the two numbers and obtain a new number. They can then capture the corresponding 'house' on the game board.

- *Chain cards*: These are a set of cards where a question is written in the lower half of the card, and the answer to a question – not the one in the lower half, but a different question, which appears on another card – is written in the upper half. The cards are distributed to different children. One child stands up and reads the question on his card. The answer to this question is on another card. So the child who has the card must stand up and read out the answer and the question that appears on *her* card. The chain goes on till it comes back to the first card. (For more details, see Subramaniam, 2001, pp. **)
- *Tic-tac-multiply*: This is a version of the common dots and crosses (tic-tac-toe). But here there are constraints on where one can put a dot or a cross. This is decided by a 'factor board' where certain numbers are written. One must choose one of the numbers and multiply it with the number already chosen by the opponent to capture a house on which the product number is written (Bhat and Subramaniam, 1997).

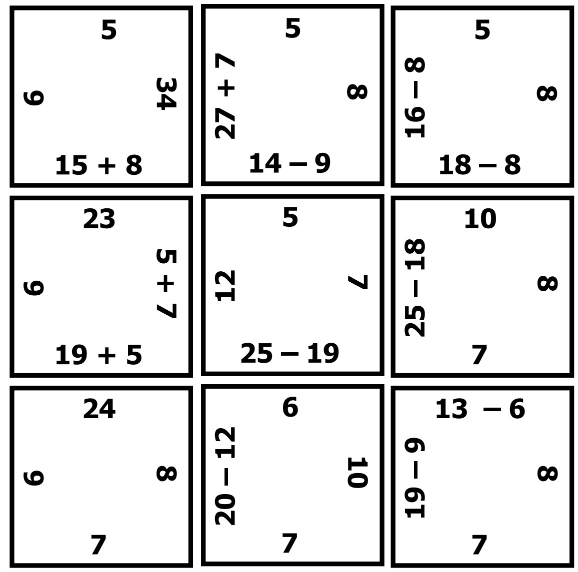

Figure 4: 'Jigsaw' puzzle

- *"Jigsaw puzzle"*: Nine squares must be arranged to form a bigger square, but the expressions and numbers along the joined edges must be equal (see Figure 4).
- *Missing area puzzle*: This is a very popular puzzle. One cuts square with area 64 units into four pieces. On rearranging the pieces to obtain a rectangle, one finds that the area is now 65 units! The children have to explain how this happens. The puzzle is helpful in securing the concept of conservation of area, a concept that Piaget showed was surprisingly difficult for children even at the end of primary school.

Some aids such as the geoboard are classic and mandatory, since they have broad application and can be used to set up puzzles or as aids to clarify concepts. Similarly the pegboard is useful in explaining the difference between area and perimeter.

Pattern based exercises

These are exercises that take advantage of the spontaneous interest shown by children in patterns. A simple patterned exercise that young children find interesting is doubling numbers. Here they are presented with several exercises such as 14+14, 16+16. The numbers in the exercises follow some pattern, which allows children to spontaneously discover simple mental addition strategies. Other examples of patterned exercises are presented below:

- A fact such as 34+58=92 is presented. Students are then asked to find 35+58, 33+58, 33+59, etc.
- Students are asked to fill in <, > or = in the box:

35+26 ☐ 36+25

- Students are asked to study the pattern in the following multiplication problems:

$$3 \times 6 = \square \qquad 3 \times 60 = \square \qquad 3 \times 600 = \square$$

- Students make as many three-letter words (nonsense words are allowed) as possible by choosing
    - B or C for the first letter
    - O for the second letter
    - L or M for the third letter

  How many words can they make? What happens if we use A or O for the second letter? And so on.

Discovering patterns involves at least implicit reasoning. This presents the teacher with a new opportunity: asking students to verbalize their implicit reasoning. Through this exercise, children learn the use of mathematical language and gain confidence in talking about mathematical objects, properties and relations.

Exploratory activities

Exploratory activities allow a longer engagement of students with a single task or situation or a set of closely related tasks and combine elements of investigatino and discovery. An example of an activity, shown in the figure below, is making and using a *mind reader*. Children explore the patterns used in constructing the mind-reader tables and practice playing the game with friends or elders. For older children and for adults, working out the connection with the binary representation of a number is interesting.

1	9	17	25	2	10	18	26	4	12	20	28	8	12	24	28	16	20	24	28
3	11	19	27	3	11	19	27	5	13	21	29	9	13	25	29	17	21	25	29
5	13	21	29	6	14	22	30	6	14	22	30	10	14	26	30	18	22	26	30
7	15	23	31	7	15	23	31	7	15	23	31	11	15	27	31	19	23	27	31

Mind Reader: Think of a number less than 32. Tell me in which of the above tables your number appears and I'll tell you your number. (The trick is to add the first numbers – which are all powers of 2 – from only those tables where the number appears.)

Many exploratory activities are described in books and journals on mathematics education. A few of them that we have tried and found useful are working with magic squares, solving the 'tower of Hanoi' problem, making and using a balance, making standard weights, various other measurement activities, making a simple fraction chart, tesselations and dissections, Moebius strips, making solid geometrical shapes from nets, etc. Measurement related activities are especially important since measurement plays a key role in craft, engineering and science. Many children still learn measurement in a purely bookish manner – one must consider this an educational trick played on students. It is appropriate here to mention another powerful resource that shares many of the features of exploratory activities. These are the so-called *open problems*. Open problems are problems that not only have different methods of solution, but that are also capable of giving rise to new variant problems or sub-problems. The approach to teaching using open problems is described in the book *The Open Ended Approach to Problem Solving* by Becker and ** (1999).

Exploratory activities, with facilitation from an able teacher can be both fun and involve a tremendous sense of discovery. Students engage in various activities that are part of doing mathematics: defining a problem, modifying or simplifying a problem to obtain a related problem, examining different cases, observing patterns and generalizing from a pattern. Such activities, of course, need careful development and planning. There is plenty of scope for innovation on the part of educators here in terms of identifying an interesting activity, exploring what opportunities it offers for mathematical investigation and reasoning, how it relates to topics in the mathematics curriculum, breaking up a given problem into smaller tasks or problems, identifying opportunities for generalization, etc.

As the students explore the activity, the facilitator plays a key role in ensuring that students derive the most out of the activity. The art of facilitation involves a balance between making helpful suggestions and letting students work on their own. If the activities are done by different groups, a summing-up session with presentations from different groups followed by explanations from a teacher can help tie up many threads of reasoning. On the whole, exploratory activities are a powerful and under-utilized medium for learning mathematics.

Unifying representational contexts

Some representational contexts model a concept or a set of concepts so well that they are truly unifying and can be used in teaching-learning situations repeatedly at different levels. Two of the most successful examples of unifying representational contexts are the *number line* and its two-dimensional analogue the *Cartesian co-ordinate plane*. These play a pivotal role in the elucidation of many key concepts in the upper primary and the high school and even beyond. Some researchers (Streefland, Ball) have proposed the use of situations and problems of *equal sharing* as an integrating representational context for the topic of fractions. Sharing problems are meaningful and are useful in illustrating concepts such as equivalent fractions, operations with fractions and also the connections between the division operation and the fraction notation. This proposal is promising and needs further elaboration into teaching-learning sequences and instructional units. Another example, on a smaller scale, is the use of rectangles and rectangular arrays in illustrating various properties of multiplication: commutativity, multiplication by one and zero, distributivity and factorization.

Teaching-learning sequences and curricula

I do not wish to discuss the principles and practice of curriculum development here, but only wish to call attention to an important area of innovation, where more work is needed, and where the demands are very different from other areas of innovation. Since mathematics is a highly interconnected discipline, innovative resources need to be integrated, and innovative practices need to be woven together, to form a coherent and extended teaching-learning sequence. The skills and concepts that pupils have learnt earlier need to be reinforced and integrated with new and more advanced concepts. Many concepts, styles of thinking and reasoning, and attitudes to problem solving develop over an extended period of time. Hence, while a few activities do advance a child’s motivation or understanding of some concepts, the greater impact is made by the curriculum which lays down what is done day after day and year after year in mathematics.

The designing of innovative curricula also presents a new level of challenge different from the development of single activities or resources aimed at a few concepts. The curriculum designer needs to worry about the overall achievement level of children at the end of the year, or even at the end of a few years of schooling. He or she has to address not only issues of learning, but also of retention over a long period. At the level of the whole curriculum, the variety of competencies and skills that make up mathematical ability become salient as also their connections with one another. The relation between procedural and conceptual knowledge, for example, needs to be squarely addressed as the curriculum designer attempts to strike a balance between different kinds of learning outcomes.

The institutional mechanism through which curricula are developed and implemented is different in different countries. In India, like in some other Asian countries, and unlike in many Western countries, curriculum development is a highly centralized activity. Even in such a context, there is a need for research and development work focussed on curriculum innovations. If such innovations are not developed and tested from within the cultural context, there is a danger of arbitrarily adopting changes, often due to the pressures of changing educational fashions. The ambience and culture of educational work must be capable of fostering within itself such research and development activity. In education on the whole, conservatism is preferable to free flowing radicalism, but it needs to be a democratic and not a hegemonic conservatism. The conservative positions underlying traditional curricula must be articulated and defended. They need to be open to criticism and fresh ideas. This means, to begin with, recognizing the importance and the necessity of alternative approaches and innovative curricula, which need space for their trial and development.

In a centralized system of curriculum development, a well articulated set of learning standards may not be perceived as a pressing need. However, even in such a system, it is necessary to specify detailed learning outcomes to address the problem of wide disparity in performance among different groups of students. Making textbooks the sole vehicle of standards leads to an over-dependence on the textbook, resulting in an inflexibility in teaching practice. Further, it is not entirely clear if the textbook creating mechanism in many states is itself guided by detailed learning standards. Another reason why debate about learning standards is necessary is the need to continuously review the place and the goals of mathematics education in the school and how these translate into sub-goals. Reflection on these issues is an integral part of curriculum development.

I have said earlier that teachers form the frontline of innovation in education. Their participation needs to be actively engaged in curriculum development. At present, this role is largely confined to giving limited feedback on textbook drafts. This only allows the collection of opinions; it does not foster innovation and improvement. Teachers need to be actively involved in innovative design of small units and parts of the curriculum. What one needs is a mechanism where groups of teachers can make important contributions at a detailed level and these contributions can be brought together and shared with a wider group. The role of teachers in developing and fostering innovative practice will form the theme of the following section.

The practice of teaching and the professional development of teachers

It is widely known that even the simplest materials can become powerful teaching and learning aids in the hands of a skillful teacher, while the most carefully designed aid can become completely ineffective if used unimaginatively. What makes up 'pedagogic skill'? Is there a way to build such skill? The second question is more relevant from the point of view of fostering improvement in educational practice. It is this question that we shall address in this section.

A well-taught mathematics lesson is nothing short of an involved performance. This performance can be demanding in terms of the teacher's skill, her alertness, her preparation, her understanding of the subject, her knowledge of the abilities of individual students and her ability to think on her feet. For a teacher burdened with several periods of teaching in different subjects everyday, it is unrealistic to expect every lesson to resemble a performance. It is important however to think of key lessons in these terms and to devote thought and effort in polishing them.

The book *The Teaching Gap*, published in 1999, reports the result of a comparative study of mathematics teaching practices in three different countries: the U.S., Germany and Japan. A large number of lessons in different types of schools in these countries were videotaped and carefully studied to understand the factors that contribute to making a lesson successful. The study was a part of the Third International Mathematics and Science Study (TIMMS), carried out in over forty countries to compare the mathematics and science achievement among students. What is most interesting and pertinent to the issue that we are discussing in this section is the practice among Japanese mathematics teachers of *lesson study*.

Lesson study is a research and development activity focused on improvement of classroom teaching through the development of a *research lesson*. It is reported to be a widely prevalent practice in Japanese elementary schools, with one teacher quoted as saying, "you won't find a school without research lessons". The development of research lessons is pursued collaboratively by teachers over a long period spread through the school year involving a cycle of design, implementation, testing and improvement. The research lessons that will be developed in a particular year are decided at the beginning of the year because they represent key topics or concepts or are linked to new educational initiatives at the school or national level. Besides the fact that this practice is a part of the system of Japanese elementary education, what is noteworthy is the collaborative aspect of the practice and the attention to the details of teaching that it embodies.

The account of the development of a research lesson in *The Teaching Gap* is fascinating. A group of teachers first meet and plan the lesson in great detail over many hours during the first trimester of the school year. The lesson is then taught to students by one of the teachers, while the other teachers in the group are present. At the end of day on which the lesson is taught teachers meet to discuss the lesson, exploring what was successful and what was not. The lesson is then improved and taught again to a different group of students by either the same teacher or a different teacher. The second time all the teachers in the school faculty attend the lesson and participate in a meeting where the lesson is again discussed and analysed. The results of the lesson study are then written up, and sometimes published, in the form of a report that can be used by other teachers.

The discussion, during the planning of the lesson as well as its evaluation often centres around very fine details as indicated by the following abbreviated list taken from the book,

- The exact wording and the numbers used in the problem which will be the starting point of the lesson
- Anticipating the solutions, thoughts and other responses of the students as they struggle with the problem
- The questions that could be asked to the students to help them at this stage to promote their

thinking

- How to use the blackboard space
- How to handle differences in abilities among students
- The ending of the lesson – considered a key moment to advance students' understanding

When I first read about the practice of lesson study, its tremendous significance in our own context struck me. One of the great banes of our system is that teachers hardly have an opportunity to learn from each other or to improve their skills by working together. The practice of sitting in on a colleague's lessons is rare. As a consequence, most teachers whether they are experienced or are novices, do not see enough examples of good teaching. In a complex skill like teaching, observing examples of actual teaching is one of the most effective ways of improving teaching. Observation must further be supported by detailed discussion of the fine points of the lesson and of teaching. This not only enables the sharing of ideas and experience, but also contributes to building a shared discourse about the practice of teaching, which as the authors of *The Teaching Gap* point out, is a necessary condition for the professionalization of the practice of teaching.

An opportunity to try out our own version of lesson study arose during a resource person training camp that we conducted for municipal primary school teachers in Mumbai as part of the PRISM (Primary Science and Mathematics Project). During this camp, teachers together planned lessons in science and in mathematics, which they taught to a group of students. The pattern followed was a day of meeting and discussion followed by a day of teaching. The lessons were videotaped and the lessons of the previous day were available for viewing on the day of planning and discussion. This intensive workshop involved 24 hours of classroom teaching of science and mathematics to a group of class 3 Marathi medium and class 4 Urdu medium students.

All the teachers were present for all the lessons. The teachers were largely from Marathi, Urdu and Hindi medium schools. After a session of teaching which consisted of one hour of science and one hour of mathematics, the teachers discussed the lesson that they had just witnessed. The co-ordinators constantly strove, through moderation and intervention during the discussion, to maintain an atmosphere where criticism would be made and received in the right spirit. In the beginning, teachers viewed the training as being similar to the micro-teaching done during B.Ed. training. There was a tendency not to focus on actual learning taking place among students. However, as the training progressed, teachers began to focus on the issue of what the children had actually gained during the lesson. One indicator of this shift was the perceived need to conduct short assessments of children's learning during or at the end of the lesson.

For all the participating teachers the training camp was an intensive learning experience. The co-ordinators felt that the teaching and the discussions greatly enhanced the confidence and expertise of the resource group of teachers as was evident from the subsequent teacher training sessions that the resource group conducted. One independent reviewer of the project who visited participating and non-participating schools in the project formed a positive impression of its impact, especially on the resource group of teachers. For us, this experience showed that generating video documentation of classroom teaching is both feasible and important. Since then HBCSE has initiated a project of generating example lessons on video.

Conclusion

I have attempted, in this paper, to communicate a sense of the variety of pedagogical resources that are available and the rationale that underlies their development and use. The point that I have urged is that the mere availability of these resources will not translate into enhanced learning for our students. The process of sharing ideas, of developing resources in a collaborative manner, and the mechanisms that enable teachers to carry this out are a great need in the present context. When this activity is undertaken in a co-ordinated manner, a shared discourse will develop around mathematics teaching which embodies a drive for innovation.

The assessment and evaluation of innovations are an organic part of the development process when the innovation is shaped through repeated trials. Carrying out trials is an essential part of development. Every practice, innovative or traditional, must be tested for measurable outcomes. One cannot understate the importance of assessment and performance measurement. However, to interpret this need in terms of an imperative to carry out controlled experiments is being simplistic. Teaching and learning, as the opening sentence of this paper says, are complex processes. Many factors are at work and these factors cannot be controlled or easily manipulated. The success of a particular practice depends critically on the teacher's appreciation of the rationale for the practice, and her skill in effecting it in the classroom. When an innovation is tested through repeated teaching trials, a 'lore' builds up about the details of its implementation. So a well-articulated innovation comes bundled in a lore that specifies how it could be implemented. When this lore is communicated and understood, it becomes incorporated into the collective wisdom of a professional group. It is this possibility that we need to provide for in building a culture that is truly innovative.

References

Ball, D. L. 1993 Halves, pieces, and twoths: Constructing representational contexts in teaching fractions. In T. Carpenter, E. Fennema, & T. Romberg, (Eds.), *Rational numbers: An integration of research* (pp. 157-196). Hillsdale, NJ: Erlbaum

Becker, J. and Shimada, S. 1997 *The open-ended approach: A new proposal for teaching mathematics*. Reston,VA: NCTM.

Bhat K.M. and Subramaniam K. 1997 *Activities for the Mathematics Laboratory*. Technical Report no. 37, HBCSE

Streefland, L. 1993. Fractions. A Realistic Approach. In T.R. Carpenter, E. Fennema & Th. A. Romberg (Eds.), *Rational Numbers, An Integration of Research* Hillsdale NY: Lawrence Erlbaum Publishers. (pp. 289-327).

Subramaniam K. 2001 *Maths for every child*. Text-cum-workbook for class 3, HBCSE.

Stigler J. and Hiebert J. 1999 *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*, Free Press