'Term' as a Bridge Concept between Algebra and Arithmetic

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Algebra has been an area of difficulty for most school students. Herscovics and Linchevski (1994) have pointed to the 'cognitive gap' between arithmetic and algebra. Researchers have attributed students' difficulties in algebra to the lack of understanding of the letter/ variable (Kuchemann, 1981; Booth, 1984; MacGregor and Stacey, 1997) and algebraic expression. Another reason for students' difficulty is that they cannot easily grasp the process-product duality inherent in algebraic expressions, that is, the fact that the expression stands for a number as well as for instructions to perform operations on the number or letter (Sfard, 1991; Tall, 1999). Linchevski and Livneh (1999) have found connections in students' understanding of arithmetic and algebraic expressions as students tend to make the same kind of errors in both places. Various studies by researchers (Kieran, 1989, Chaiklin and Lesgold, 1984) have claimed that most students in the elementary grades are not aware of the underlying structure of arithmetic expressions. They do not understand for example, that $683 - 297 + 235$ and $235 + 683 - 297$ are equal without calculating.

Research in the area of teaching and learning of algebra has indicated the importance of understanding the structure of arithmetic expressions to make sense of algebraic expressions and their manipulation. However, recently Linchevski and Livneh (1999) have raised doubts about whether structure oriented arithmetic teaching is really appropriate as a preparation for algebra.

A study, being conducted at HBCSE, Mumbai, aims to look at the transition from arithmetic to algebra in beginning algebra students (class VI). The transition takes place in the context of a teaching intervention, focussing on the structure of arithmetic expressions, and thereby exploiting the structure sense in understanding and manipulating algebraic expressions. Three phases of the study have been conducted with students from nearby English and vernacular (Marathi) medium schools. The first phase was exploratory and aimed at developing instructional material. In the second phase, a two-group design was formulated with one group receiving algebra instruction with instruction in arithmetic and the other group receiving algebra instruction without arithmetic. In the third phase of the study, a single group design was used with two groups of students studying in English medium and one group of Marathi medium students. There was no control group with all the groups being subjected to the intervention. Pre and post-tests were given to see the improvement in performance in each of the phases.

During this study, the concept of *term* has been identified as crucial for understanding the structure of expressions as well as for making the transition to algebra. Term is a number with the $+$ or $-$ sign preceding it, attached to it. For example, the terms in the expression $12 + 4 - 5$ are $+12$, $+4$ and -5. In the traditional textbooks, terms are introduced in the context of algebra in class VI, and like and unlike terms are subsequently defined, leading to the rules for manipulating and simplifying algebraic expressions. The concept of term is neither connected to any other concept like the concept of equality nor given any meaning like $+4$ is '4 more' and -5 is '5 less'. The study shows that the idea of term is very powerful. It becomes impoverished by restricting its use to merely introducing rules of syntactic manipulation. In this paper, we discuss contexts

where the concept can be of help to students not only in perceiving the structure of arithmetic expressions but also in seeing the parallels between arithmetic and algebraic expressions.

In the instructional sequence used for the teaching study, students first learnt to parse arithmetic expressions correctly using terms. A distinction was made between simple terms like $+2$ or -3 and complex product terms like $+3\times5$. In the first phase, the rules of order of operations and the idea of terms were kept distinct, whereas in the later phases they were combined to give a more unifying approach as well as to give meaning to the rules of order of operations. Students are usually taught the convention of order of operations through rules such as 'do multiplication before addition and subtraction'. Now the focus was shifted from this procedural way of looking at an expression to a structural way by emphasizing that in order to evaluate an expression the terms in the expression must be combined (Subramaniam, 2004). Simple terms can be combined easily and a product term and a simple term cannot be combined unless the product term is converted into a simple term. While a pre-post comparison of students' performance indicated that they readily learnt this approach to evaluating expressions, the retention of these concepts over a period of time was low as is seen in their performance in the delayed post-test. It is possible that doing these exercises in the school in the traditional way interferes with the alternative way learnt for a very short period with no further reinforcement.

Students applied the idea of terms to (a) generate arithmetic expressions equal to a given expression (involving both simple and product terms) by rearranging terms and (b) to judge whether two expressions are equal. The former group of tasks were open-ended in the sense of having multiple correct answers. Students performed well in both groups of the tasks and achieved an accuracy of 60% to 70% in judging whether two expressions are equal. After working with arithmetic expressions, students could more easily write equivalent algebraic expressions (involving both product terms and simple terms) by rearranging the terms so that like terms are together, which is the key step for simplification. Effort was made to minimise the occurrence of conjoining in algebraic expressions (writing $2x + 3 = 5x$) by reminding them of the rules of order of operations in arithmetic expressions. The students stated these reasons in classroom discussions and would point out the mistake done by their colleagues but this did not always translate to correct performance in the written tests. Still, the students who had undergone such instruction performed better than those who had not, with the frequency of conjoining in the former group being around 15% to 20% and for the later group being around 45%.

The data collected through video recordings, classroom observation, daily practice exercises and tests suggest that clear understanding about terms can enable the students to appreciate the structure of the expressions and also help them to make a transition to algebra from arithmetic.

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