'Term' as a bridge concept between arithmetic and algebra

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Arithmetic and Algebra: A complex relationship

Herscovics and Linchevski (1994) pointed out the existence of 'cognitive gap' between arithmetic and algebra.

Linchevski and Livneh (1999) raised doubts over the appropriateness of teaching structure oriented arithmetic as a preparation for algebra

Terms

- Term is a number with the preceding + or – sign attached to it.

  Terms of 12 + 5 – 3 are +12, +5 and -3.
- Crucial for understanding the structure of expressions
- Can help in making the transition from arithmetic to algebra

Use of terms in Indian curriculum

Introduced in grade 6

Like and unlike terms defined

No other meaning and no connection with concepts like equality

Impoverishing the powerful idea by restricting its use to syntactic manipulation

Aim of the study

- To develop general principles for organizing the teaching of symbolic algebra
- To relate these to current research on algebra learning
- To identify good tasks
- To trace the development of students' understanding about symbols and expressions

Methodology

- A design experiment with iterative teaching cycles
- Multiple groups across cycle
- Four cycles from April 2003 to Nov 2004
- Grade 6 students
- Groups studying in English and in Marathi

The use of the concept of ‘term’ in different contexts and tasks

Evaluation/ simplification of arithmetic/ algebraic expressions

Examples: 4 + 6 × 8, 15 – 3 – 7, 13 × y – 4 + 5 – 6 × y

Concept of ‘*term*’

Two kinds of terms: ***Simple term*** (+ 3) and ***Product term*** (– 2 × 3)

Simple terms can be combined

Product term and simple term can be combined by converting the product term to simple term

Product terms can be combined if they have a common factor

Equality of expressions

- *Generating equal expressions* (like, 23 – 16 + 18 or 15 × 8 + 17 – 6) by rearranging terms, splitting terms as sum, difference, product, compensating terms
- *Identifying expressions equal* to a given expression from a list of expressions
- *Showing an expression to be equal to another* by manipulating one expression (48 – 23 + 12 – 17 = 60 – 40, 19 × n – 8 – 5 × n + 1 = 7 × (2 × n – 1))

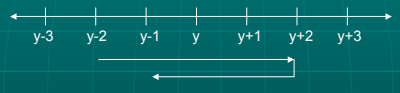
Bracket opening rules

- Bracket term introduced
- Equality of expressions made the basis of the rules
- Inverse of an expression introduced
- Verbal explanations given whenever needed

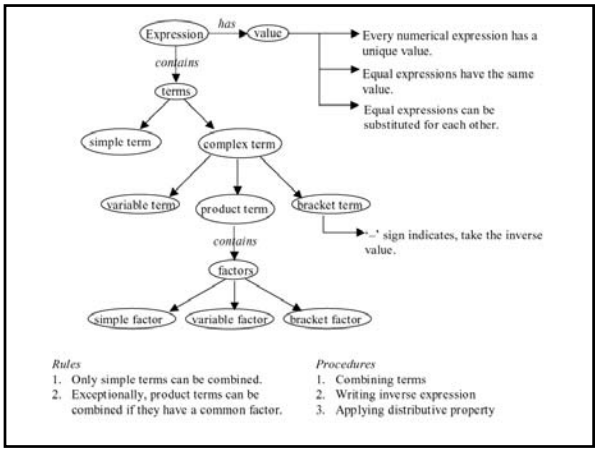
Integer operations using terms

- Subtraction same as adding the inverse

- *Writing an expression* for number line journeys



- *Distance between two points* on the number line
- Exercise on *evaluating expressions by easy ways* (-28 + 49 + 8 + 20 – 49, 14 × 3 + 10 × 8 + 14 × 7)



Results

Structure of expressions

- Identifying terms in an expression: 80% to 100%
- Generating expressions equal to a given expression:

  Classroom data good (e.g. 3 – 5 + 2 × 7 = 3 – 2 – 3 + 2 × 7)
- Identifying an expression equal to a given one from a list of expressions:

  60% to 70% for expressions obtained by rearranging terms

  Between 50% to 80% for expressions involving splitting of terms or using brackets or compensating terms.

  About 30% correct responses for more difficult questions

Performance across phases for bracket opening tasks

Identify an expression equal to 18 – (7 + 5)	April 2004	Nov 2004
English	25%	57%
Marathi	71%	82%

- Across all bracket opening tasks with + and – to the left: English around 45%, Marathi up from 57% to 70%

Syntactic manipulation:

- Evaluating expressions (e.g. 8 + 5 × 7, 13 – 6 – 3): 90% and above compared to 70% to 80% in earlier phases
- Finding easy ways to evaluate long expressions (e.g. -28 + 49 + 8 + 20 – 49) : 80% compared to 50% earlier
- In many instances students used distributive property to make evaluation easier

Task: Evaluating expression by easy ways	English	Marathi
11×4+9×11–7×11 (one factor common)	55	72
12×9+16×5–17×9 (distributivity twice)	14	16
m+15–13×m–9 (variable factor)	0	20

Simplifying algebraic expressions: 50% of English medium and Marathi medium (3rd phase), higher than 2nd phase 25% for these groups

Conjoining error: 15% to 29% for different groups

Letter-number-line journey: 64% English medium and 23% Marathi medium

In some tasks, e.g. finding distance on the letter-number-line, performance has dropped (30% to 15% for English medium and 55% to 43% for Marathi medium) with some new errors appearing

Evaluating algebraic expressions: English medium 52% to 62% and Marathi medium 85% to 93%

Conclusion

The data suggests that:

- An approach using 'terms' as a key concept helps students see the parallels in the structure of arithmetic and algebraic expressions
- It allows the students to use their arithmetic sense as a spring-board for algebra learning

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