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Understanding teachers' concerns and negotiating goals for teaching: Insights from collaborative lesson planning

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***Abstract**: In this paper we describe our insights from an intervention with middle school teachers aimed at collaboratively planning a lesson for teaching the topic of integers. The collaboration involved teachers working together with teacher educators to share teaching approaches, explore different meanings of integers, explore contexts and develop resources for teaching integers. It led to opportunities for teachers for negotiation of beliefs, developing shared values of making mathematics accessible to students using close to real life contexts and opportunities for going beyond the textbook by making decisions on the basis of one's own knowledge.*

***Key Words:** Teacher-researcher Collaboration, Lesson planning, Professional development, Integers, Real life contexts*

INTRODUCTION

Lesson planning is an integral part of teaching. In India, mathematics teaching is textbook driven ([National Council of Education Research and Training [NCERT], 2006, p.15) and the teaching approach and exercises that teachers include in their lesson plans are largely taken from the textbook. Thus, in the received view of teaching, teachers are not required to think explicitly about most teaching decisions. This results at times in teachers failing to understand the rationale behind the approaches and at worst in teachers misinterpreting them as mechanical sequence of steps to get correct answers. Thus going beyond the textbook is an important goal for reforming mathematics teaching (NCERT, 2005).

For any change that is visualized for classroom teaching, teachers need to be provided with opportunities to negotiate beliefs as well as to acquire knowledge that empowers them to make informed decisions in the classrooms. Lesson plans can serve as “concrete scaffolds for teachers to focus their attention and learn about specific content area under discussion” (Hart, Alston & Murata, 2011, p.7). Research related to teachers' learning through collaborative lesson planning mostly comes from lesson study research. For e.g., it has been found that collaboratively anticipating and discussing student thinking while making lesson plans leads to teacher learning and that lack of subject matter knowledge and reasoning skills constrain the opportunities for learning during lesson study (Fernandez, 2005). Meyer and Wilkerson (2011) elaborate on how opportunity to develop teachers' knowledge arise through the discussion of concepts and instructional strategies prior to making a lesson plan than the use of existing lesson plan and focusing on its implementation. The role of facilitator has also

been identified as important in providing guiding questions and eliciting teachers' pedagogical content knowledge which had not been formally shared before (Hart and Carriere, 2011).

The literature related to teaching integers can be categorised as advocating algebraic approach and modelling approach to instruction (Linchevski & Williams 1999). While Fishbein (1987) argues against use of “out of school” meaning to negative numbers, models like double abacus (Linchevski & Williams, 1999), expressions of movement of lift (Ball, 1993) have been shown to be useful in developing students' reasoning about integers. The algebraic approach focuses on treating expressions as generalised arithmematic as compared to approach using models where attempts are made to “concretise” rules for integer operations throgh use of contexts. However we feel that reasearch is needed to elucidate how teachers make a selection of approaches and its relation to teachers' knowledge about integers and beliefs about teaching mathematics.

In this paper we present an analysis of the episodes of interaction that occurred in a workshop on collaborative lesson planning to illustrate the connection between teachers' thinking about integers and issues related to teaching integers. Through this analysis we highlight the opportunities for learning that arise during lesson planning for both teachers as well as teacher educators.

THEORETICAL FRAMEWORK

We view professional development as providing opportunities for teachers to engage with their knowledge and beliefs about the various aspects of mathematics teaching within the community comprising teachers, researchers and teacher educators. Teachers' growth is thus organically connected to the growth of knowledge to bridge the divide between research and practice on the one hand and between practice and theorisation of teaching on the other. This can happen when teachers engage with opportunities to learn through closely analysing, making hypothesis and reflecting on the work of teaching. Articulating beliefs and sharing knowledge comes naturally for teachers when discussing the work of teaching when it is made evident to teachers that they are valued by members of the community. Although these activities are in some ways inherent to the everyday work of teaching there are some added advantages in doing tasks situated in teaching in a community comprising of teachers and teacher educators. These advantages can be in form of reflection on one's own experiences and making them available as *knowledge* to the community, while also being challenged to establish this knowledge as a member of the community. Knowledge and beliefs of all members contribute to the growth of knowledge and beliefs as a community. Thus we do not view the flow of ideas as unidirectional from teacher educators to teachers but as multi-directional, affecting all those who are participating. In the case of in-service professional development, we found this idea especially fruitful since it involves working with teachers who have already taught for several years and hence possess situated knowledge about students, about the mathematics they teach, as well as knowledge of the context in which they are teaching although such knowledge may not be apparent or valued. Dealing with teachers' knowledge and beliefs respectfully would mean providing them

opportunities for sharing their approaches, acknowledging and building on their knowledge and if needed respectfully challenging them as a member of community, while expecting the same from teachers when ideas are shared by other teachers or teacher educators. The shared values adopted by community members play an important role in guiding discussions and learning that takes place as a result. Thus we view ***Situatedness***, ***Challenge*** and ***Community*** as three aspects of professional development through which opportunities are provided to teachers to revise, reflect upon and rebuild the knowledge and beliefs held by them.

THE STUDY

We undertook a study involving in-service teachers in collaborative lesson planning through 6 one day workshops spanning a period of four months. This was part of a larger study aimed at developing a model for professional development of mathematics teachers. Eight teachers teaching in local schools belonging to a large government school system participated in the study. Previously, these teachers along with other teachers had attended a ten day professional development workshop with the research group a few months before collaborative lesson planning workshops took place. The teachers were divided into two groups, a group of four primary teachers and a second group of four middle school teachers. During the workshops there were some common sessions where both groups discussed and shared what they were doing. During this period the first author also did participant observation of the classroom teaching of one primary and two middle school. In this paper the focus on the middle school teachers' group and the discussions that took place during the lesson planning for the topic of integers.

During the 6 one day workshops teachers engaged in various activities before actually developing a lesson plan. Due to paucity of space we provide a brief description of those activities here. On the first day teachers were asked to select a topic for the study based on what they consider as difficult for students. Teachers selected the topic of integers for the sixth grade to work on since they felt that students face problems in understanding operations with integers. They discussed their teaching approaches, students' common mistakes and ways of resolving them, the situations and representations used by them. The teachers were then engaged in the task of describing situations where integers would be used in different senses, that is, to denote “State, Change and Relation” (Vergnaud, 1982). This was followed by discussing and listing of learning outcomes for the topic of integers and analysis of the textbook chapter. Next, teachers engaged in the task of framing and categorising questions for addition and subtraction of whole numbers as well as integers into the categories of 'Combine', 'Change' and 'Compare'. Teachers then developed activities, questions and resources for developing understanding of integers addition and subtraction in the three different senses. One of the contexts used was that of a lift in a building for which teachers along with teacher educators framed questions for students to mathematically represent situations and discussed the appropriate sequence of questions. A poster showing the lift was made for teachers to use in their classroom along with a suggested sequence of questions. Teachers then developed and discussed their individual lesson plans after incorporating insights from the exercise. After teaching in their respective classrooms, teachers shared their experiences of teaching with each other and with a new group of teachers.

The data was collected in the form of audio recording of all the sessions as well as artifacts like teachers' notes, lesson plans, of which copies were made. Audio files were later scanned by the researcher while making notes about the discussions. Selected segments were transcribed and analysed further by making concept maps which were used to arrive at findings. The methodology borrows from the qualitative data analysis advised by Miles and Huberman (1994) for developing emergent pattern codes from the data and using data displays to show relationship between codes.

FINDINGS

We describe below how opportunities for teacher learning arose while establishing goals, discussing students' common mistakes and exploring pedagogical approaches and contexts.

Addressing common mistakes: A window into students' thinking

In the context of lesson planning, teachers shared students' common mistakes in operation with integers. In the first few sessions the most common reason that teachers gave when asked to explain why students make mistakes was that "students know the concept but they forget it later". This suggests that the teachers viewed learning of mathematics as memorizing rules or procedures which students later forget. Still, there were differences among teachers in the emphasis they placed on teaching rules. This was also reflected in the common mistakes of students that they considered as important to address. For e.g., one teacher felt that it is important to address the common mistake of reversing the order of numbers while subtracting when solving the question "subtract 7 from 3". According to her, the error was because student misunderstood the language and student has to "keep in mind" that she has to reverse the order of numbers while writing in numerical form (3 - 7). Another teacher's concern was to address the common mistake that students when asked to solve "3 + (-7)" write -10 "since they will add first and then will put the minus sign".

Teachers' sharing of common mistakes made by students provided teacher educators with an idea about the concerns of the teachers and to create opportunities for teachers' learning by extending discussions about students' mistakes to discussions about student thinking. In the above case, the teacher educator asked teachers to think of what the word "subtract" might mean to students. Teachers were able to come up with the meanings that students might associate with subtraction like "minus sign", "finding the difference" and "giving /taking" that students come across in primary mathematics. This realisation helped them to develop alternative explanations of student mistake that subtraction of smaller number from bigger number might appear meaningless to students while having "take away" meaning thus resulting in the mistake of reversing the order for the problem to make sense.

The facilitator responded to the second mistake by asking teachers that "why do students make such mistakes?". In the course of discussion the teacher who shared the mistake realised that mistake was due to students "viewing the signs + (plus) and - (minus) as operations and not as signed numbers". This was followed by discussion about whether one should read the problem 3 - (-4) as "3 minus negative 4" to make the distinction clear for students. This discussion was revisited when a teacher shared the approach using number line using the notion "Minus meaning change in direction". It would involve changing the direction twice

when solving for e.g., 2 - (-6). Some members felt that students can interpret it wrongly while solving -3 - 4 and will change the direction for -4. It became evident that the distinction between the minus sign used as operation and as signed number is not focused in some pedagogical approaches.

The discussion about student mistakes was revisited when meeting with primary teachers group. A primary teacher reflected that such mistakes of reversing order could be due to undue emphasis on telling students to “Always subtract the bigger number from the smaller number” in primary classes. Another primary teacher objected to this as she felt it would be difficult to explain the concept of “borrowing” in subtraction if they don't use that rule. This provided an opportunity for the teacher educator to talk about how “regrouping” can be used to explain the subtraction procedure and would also make the mathematics consistent for student across grades. Thus discussion on students' mistakes provided learning opportunities to both middle grade and primary teachers for their approach of teaching subtraction and developing connections within mathematics across grades.

The discussions surrounding student mistakes generated much discussion on student thinking and over the course of the workshops a shared value of evaluating pedagogical approaches for their accessibility to students had developed. This was evident in the kind of arguments that teachers made while evaluating pedagogical approaches. For e.g., a teacher considering that subtraction of integers is difficult for students suggested changing the subtraction problem of integers into addition as given in the textbook and then using the rule for addition i.e. convert the problem “-6 - 4” into (-6) +(-4). Another teacher objected to this on the ground that it would not make sense to students as to why one should convert it into addition problem. In another instance while discussing an approach a teacher commented “ this gives correct answer but if a student asks 'why' then we will have to explain”. Thus teachers evaluated pedagogical approaches by anticipating students' doubts and questions and whether mathematically correct answers existed for such questions.

The differences amongst teachers in identification and interpretation of common mistakes by students indicated differences that existed in varying degrees in teachers' pedagogical content knowledge as well as the beliefs about teaching mathematics. Some teachers recommended procedures that they expected students to remember while others tried to interpret students' difficulties indicating their sensitivity to student thinking. The teacher educator's questions challenged the teachers to look at the symbol of minus as a student experiences it when introduced to integers and the way she tries to make sense of it by using her knowledge of subtraction. Teachers realised that the distinction between subtraction as an operation and a negative integer is missing in some pedagogical approaches and may underlie student mistakes. There was a shift in teachers' discourse for explaining students' mistakes by connecting them to teaching approaches, students previous knowledge and the possible interpretations that students can construct, instead of attributing it to students' faulty memory as done initially. Also there was a realisation of how connections need to be established in mathematics taught across grades to help students make sense of mathematics.

Tension between using rules and using models for teaching integers

The review of literature for instructional approaches suggested for integers shows the dichotomy between algebraic approaches and approaches using models. Historically also integters were theoretically founded formally through algebraic laws although attempts were made to find congruences in real life. During collaborative planning these dichotomies were evident as tensions between using rules versus using models. On the one hand teachers felt that use of models is not useful for teaching integers since “students ultimately need to know the rules to solve integers problem” and as “ representations cannot be used to solve integers problem with large numbers”. On the other hand they recognised the students' difficulties in making sense of negative numbers and thus suggested using situations like borrowing, increase and decrease of levels. However the teachers admitted that use of situations was limited to introduction of the concept of integers.

This tension between being honest to the discipline and encouraging student sense making was evident in one of the questions that a teacher asked and the discussion that followed after it. A teacher asked the teacher educator “How one can convey to a student that +1 and -1 together becomes a zero”. Mathematically justification of +1 and -1 becoming zero together is through use of axioms but the problem presented here was to make it meaningful for students. The teacher educator showed how an activity with buttons having two different colors can be used to develop meaning of plus and minus sign as opposite to each other and thus cancelling each other to get zero. Teachers explored how subtraction of smaller number by bigger number can be valid in the context of this activity by introducing zeroes in form of pair of “+1 and -1”. Teachers explored other contexts like borrowing and increase/decrease in quantity which can be used to convey this idea. The teacher educator then further developed this idea by linking the concept of additive inverse with the equivalence of subtraction and addition of the inverse.

Another instance depicting these underlying tensions was when a teacher challenged the approach of converting the subtraction problem of integers to an additon one (discussed in previous section) by asking for an explanation for why only the minus sign in the second number is changed to addition and not the first number in for e.g., -5 - 3. Here teacher is using the *knowledge about mathematics* to challenge the logical consistency of the pedagogical approach since mathematics has to be consistent.

There was evidence that one of the teachers' understanding of mathematics discipline as a deductive axiomatic structure conflicted with the use of models for teaching mathematics and thus he rejected them over using rules as “laws” of mathematics. This teacher's lesson plan focused on telling students the rules in the form of “laws”, and having them practice the application of the laws in numerical problems after discussing a few situations (using integers to represent the amount borrowed and to represent distances above and below sea level) in the introduction. The teacher's sensitivity towards logical consistency was a resource in evaluating pedagogical approaches by emphasising the value of the properties of mathematical structures as embodied in the approaches discussed. This indicated way in which subject matter knowledge interacted with pedagogical content knowledge.

Developing teachers autonomy: relying on textbooks to making decisions on their own

In initial discussions it was apparent that teachers were hesitant about using approaches that were not given in the textbook as they resisted engagement with such approaches. Even when teacher educator shared approaches, teachers said that they "won't be able to use them in the classroom since in the examinations students are required to solve problem by a particular method". Teachers engaged in the process of analyzing the textbook and discussed the approaches given in it thus deepening their understanding about them. It made explicit how teachers operationalised the approaches in the textbook in classrooms. While sharing the approaches from the textbook (refer discussion changing subtraction problem to addition in previous section) teachers seemed to have a superficial understanding of approaches given. The textbook used the approach of converting subtraction to addition problem by using a missing addend problem while the teacher explained the approach in terms of converting every subtraction problem into addition by changing the sign of subtraction operation between two integers. The facilitator provided an example of how subtraction can be interpreted as addition by using "count up" strategy. e.g. difference between 100 and 45 can be found by finding what number needs to added to 45 to get 100. Thus discussing textbook approaches provided opportunities to move beyond understanding of rules as manipulation of symbols to using algebra to make sense of rules.

Discussion of teaching aids, approaches, questions with teachers had a rich variety of notions that teachers used in their talk about integers like position, change in quantity, direction, movement, positive and negative integers as opposites. A teacher discussed how "-2 can be referred not only with zero as reference point for e.g., to denote temperature but also for denoting 2 degree decrease in temperature with a flexible reference point". However teachers mostly discussed these notions as unconnected examples or questions and did not explicitly talk about using them for for teaching. These notions served as resources for teachers to build their knowledge about integers and their operations when they later engaged in activities like identifying and exploring contexts where it made sense to use integers as "state, change and relation" (Vergnaud 1982). The three senses not only helped teachers in understanding how different examples are related but also helped in evaluating the questions given in the textbook. The analysis of the chapter on "Integers" in the textbook by teachers and teacher educators together revealed that the questions given in the textbook catered mostly to the sense of "state". This provided some motivation for teachers to explore other meanings of integers and to engage with approaches that cater to variety of senses. It helped teachers in building a theoretical basis for consolidating their pedagogical content knowledge thus making it readily available for decision making.

The evidence of teachers developing autonomy from textbook came from their exploration of approaches to build on students' previous knowledge and that which encouraged sense making. For e.g., a teacher thought using the "take away" meaning of subtraction through an activity. She developed an activity by representing the number of stones removed and added to a bowl (containing unknown number of stones) mathematically as integers.

Exploring contexts to mathematise operations of integers : From symbolism to making meaning

In the beginning, teachers shared various approaches for teaching integers ranging from direct use of rules, to use of situations, examples and representations. Their use of representations like number line involved use of rules of thumb like minus meaning change the direction (discussed previously), going forwards for positive and backwards for negative. Teachers explanation of approaches indicated that use of situations or representations was an aid to get correct answers. Explanations were mostly about how to manipulate symbols to get the answers and rarely on how these symbols represents quantities or actions in the situations discussed.

Over the course of several explorations involving making sense of operations on integers using different contexts, 3 of the 4 teachers realised the potential of using them in the classroom. This was indicated through teachers' enthusiasm in thinking about various contexts and activities that can be used in classrooms and their incorporations of approaches using contexts in their lesson plans to make negative integers and addition and subtraction of integers. Some of the contexts explored were movement of a lift in a building with floors below ground level, growth chart of a baby's weight, temperature data, vertical positions of different objects like aeroplane, boat, submarine, etc. Out of these the context of lift was developed and used extensively by all the teachers. In developing these contexts as resources, teachers used their knowledge of these senses and situations to understand how actions performed in various contexts can be represented mathematically as numerical expressions depicting operations with integers. For e.g., movement from -2^{nd} floor to 4^{th} floor can be depicted as 4-(-2)= 6 when considering the difference between floors and also as +2+4= +6 to represent net change in position. These teachers planned for giving opportunites to students to extract rules for addition and subtraction of integers after experiences with contexts rather than introducing rules early in instruction.

Exploring several situations for teaching different senses of integers possibly helped in developing criticality for selecting appropriate situations and understanding strengths and limitations of using contexts for teaching different aspects of the concept. For e.g., after teaching the topic, teachers in the workshop sessions discussed how the situation of comparing the scores of two groups in a quiz made sense to students when negative score is given for wrong answer. Students could justify that -2 is more than -3 as the group scoring '-2' made less mistakes. In comparison while discussing temperature one says -3 degree C is colder than -2 degree C. Teachers found that it was difficult to convey that -3 is a lower temperature. Since Mumbai students have rarely or never experienced such low temperatures it added to the difficulty. Thus developing nuanced understanding of senses and the various contexts teachers were able to appreciate the role contexts play in connecting students' intuitive and informal knowledge with that of formal mathematics.

DISCUSSION AND CONCLUSION

Collaborative lesson planning involved teachers and teacher educators in working together and provided opportunities for learning for both teachers and teacher educators. The three

aspects - situatedness, challenge and community played important roles in bringing about these opportunities for teachers to learn.

The activities that teachers engaged in were situated in the work of teaching in many ways. When teachers selected the chapter for study they articulated the reasons which they thought made certain topics difficult for students. Sharing of common mistakes of students made clear the challenges that teachers faced in teaching of integers. Opportunities to share, elaborate, evaluate the approaches used as well as to analyse the approaches given in the textbook allowed teachers to make their situated knowledge explicit and situated the discussions in teaching of integers as experienced by teachers. The discussion brought forward the beliefs, tensions and dilemmas that a teacher faces when selecting approaches and how the subject matter knowledge interacts with pedagogical content knowledge. The differences that existed among teachers in terms of knowledge and beliefs challenged them to move towards deeper explanations considering students' thinking and mathematical consistency. These discussions provided teacher educator with a good idea about how teachers think and thus to think of ways to challenge certain beliefs and support knowledge development. Interestingly teachers themselves took initiative by asking questions and provided counterarguments to the approaches being shared as they adopted the values of making mathematics accessible to students. Engaging in discussions situated in one's own teaching experince thus provided teachers with the opportunity to reflect on and develop their knowledge in a community. In this intervention the presence of primary and middle school teachers together in a community led to deeper understanding of the curriculum across grades as well as helped in developing explanation for the common mistakes. The community provided site for sharing their learning with each other thus contributing to growth of knowledge about teaching integers. The teachers and researchers working together in a community contributed to bridging the research practice divide by planning the lesson together and sharing the results.We argue that it is important to develop communities comprising teachers, teacher educators and researchers where the discussions are situated in the work of teaching while providing challenges and support to facilitate building on teachers' existing knowledge. It is more likely to be more meaningful for teachers to use in their teaching and planning for teaching.

The role of teacher educators as collaborator contributed to developing shared values in the community as well as supporting teachers in building their own knowledge through design of tasks, posing challenging questions and developing a theoretical basis for understanding teaching. The teacher educator's role in the community involved both supporting function to make the knowledge and beliefs of members explicit as well as challenging function by critically analyzing the conjectures and assertions posed by the members.

The discussions with teachers revealed how teachers’ understanding of students’ common mistakes, teaching approaches and conceptual understanding of integers are intricately connected to each other. The findings indicate that by providing opportunities to teachers to deepen their content as well as pedagogical content knowledge, shifts in teacher's thinking can be initiated about the beliefs held and the pedagogical approaches. The evidences of teacher learning were in form of shifts in teachers' discourse about student thinking and mathematics teaching, evidences of development of shared values of making mathematics

accessible to students, teachers' use of nuanced understanding of different senses of integers in planning for teaching integers and finally developing critical outlook for selecting situations for teaching integers while considering students' conceptions.

Lesson planning might be important for researchers to study because it can provide a window into the teachers' thinking and decision making which takes place in the classroom. The study has implications for considering collaborative lesson planning as a tool for in-service teacher professional development by enhancing their awareness and knowledge about aspects otherwise ignored and negotiation of beliefs as a community.

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