Elementary Teachers' Beliefs and Practices for Teaching of Mathematics

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We report beliefs and practices concerning the teaching and learning of mathematics of 13 elementary teachers from a government school system in India through selected items in a questionnaire, interviews, records of discussion during a workshop and classroom observations. Responses to selected questionnaire items indicated that teachers' beliefs varied. The interview and classroom data indicated that in practice most teachers gave primacy to obtaining correct solutions through taught procedures consistent with a "transmission" view. Teachers felt that showing procedures or solution to problems and repeated practice lead to learning mathematics. Teachers believed that students cannot discover procedures on their own and that students have to be taught so as to avoid errors. The possibility of change of beliefs and practice may be strongly constrained by teachers' own mathematical understanding.

Introduction

The position paper on teaching of mathematics of the National Curriculum Framework recognises that teaching of mathematics has been textbook centered with the focus on learning mechanical procedures rather than developing students' power of mathematisation and reasoning (NCERT, 2006). The document recognises inadequate teacher preparation as one of the reasons for the prevalent approach to mathematics teaching. Studies elsewhere in the world have indicated that focus on change in teaching strategies without taking *teacher thinking* into consideration leads to teachers making superficial changes without having any significant change in student learning opportunities (Cohen, 1990). It is therefore important to first understand beliefs and practices that are prevalent among teachers in order to support reform in teaching that is not superficial.

Relation between beliefs and practices

Knowledge, beliefs and emotions have been found to play an important role in shaping teachers' thinking. Although beliefs have been considered as a messy construct (Pajares 1992) there is general agreement in the mathematics education community that mathematical beliefs are "personal philosophies and conception about the nature of mathematics and its teaching and learning" (Thompson 1992). Beliefs play an important role in the way the teacher makes decisions during classroom instruction and thus impact the understanding that students develop as a result of instruction (Wilson & Cooney, 2002). Change or development of beliefs is recognised as a difficult and long term process (Clarke, 1994; Swan, 2006). Research also suggests that belief and practice are in dialectical relationship with each other and influence each other (Guskey, 2000).

Several research studies report dissonances between teachers' beliefs about mathematics, its teaching and learning and actual classroom practices. Various explanations have been offered for such inconsistency between "articulated beliefs" and "enacted beliefs" (Ball & Even, 2009): that some beliefs are held more centrally than others (Pajares 1992), or that the constraints and supports available in the teachers' context allow teachers to enact some beliefs in consonance with their present purpose while assigning lower priority to others. We agree with Aguirre and Speer (2000) that beliefs occur in form of belief bundles that "connect particular beliefs from various aspects of entire belief system" (p.333). How certain beliefs are activated during teaching might depend on long and short term goals (Schoenfeld 2003), the context of the teaching situation, unexpected occurrences during teaching and the knowledge held by teacher to deal with the situations arising during teaching.

Some have questioned the methodologies adopted for belief attribution suggesting that researchers and teachers may have different interpretations and meanings (Speer, 2005). This points to the inherent difficulty of describing teachers' beliefs despite their centrality in influencing teachers' thinking and practice, and the need to draw on multiple sources and use mixed methodologies while ascertaining the beliefs of specific teacher groups.

In this paper we attempt to illustrate some beliefs and practices, related to mathematics and its teaching, of teachers in a government school system using data from items in a questionnaire, interviews and workshop discussions, and classroom observations. The research question addressed by the study is to identify the nature of knowledge and beliefs relevant to mathematics teaching of the teachers participating in the study. The beliefs and practices discussed have to do with emphasizing procedures, the use of repeated practice and preventing the occurrence of student errors as important goals. The four themes together constitute a belief bundle making up a view of mathematics as a subject aimed at obtaining correct solutions using known procedures. We illustrate how this belief bundle is supported by the knowledge of mathematics held by the teachers. We also share insights for the development of tools for assessing teachers' beliefs and practices.

The Study

The findings reported here are part of a larger study from 2009-2011 on collaborating with teachers to develop classroom practices aimed at teaching mathematics for understanding. The study had different phases: professional development workshops and collaborative follow-up of classroom teaching by the first author. Participants in the study were mathematics teachers teaching primary and middle grades in a nation-wide Government school system and were nominated by their principals to participate in the study. Of the 13 teachers from the system who participated in the first workshop, 4 primary and 4 middle school teachers were local, (from the same city) and participated in the two subsequent phases of the program. All the teachers who participated had more than 15 years experience of teaching and were between the age range of 39 to 50 years. Out of 8 middle school teachers only 3 were males and there was no male primary teacher.[1]

Data Collection

Data about teachers' beliefs and practices were collected through Likert type written questionnaires (balanced scale having both positive and negative statements), detailed individual interviews during the professional development workshop, written logs of classroom observation and notes of discussion during the collaborative phase. All the sessions of the workshop were video recorded. All the interviews, and most of the classroom lessons and discussions were audio-recorded. The questionnaire had six parts focusing on teachers' beliefs about mathematics, its teaching and learning, frequency of practices adopted, beliefs about self, beliefs about students, and teachers' personal data. Content validity of both questionnaire and interview was done by experts (researchers, teacher educators) and changes were incorporated as per suggestions.

Participants in the workshop including the 13 teachers from the government school system completed the questionnaire in about an hour. Each interview took approximately one hour. The interview was semi-structured in nature where prompts were provided by the researcher based on the items given in the questionnaire as well as questions about teachers' experiences with mathematics in the past as a student and as a teacher. During the collaborative phase of the study extensive classroom observations were done for 1 primary teacher and 3 middle school teachers from among the group of 13 teachers for a minimum of 20 lessons of 35 minutes each. At least one lesson was observed of the remaining 4 local teachers from the group. The researcher also recorded personal reflections about conversations with teachers, principals and headmistresses along with the circulars, inspection forms and exam papers.

Data Analysis

The Likert scale items in the questionnaire were coded and entered into a spreadsheet for further analysis. Interviews of 4 middle teachers (Teachers A, B, C and D) and 4 primary school teachers (Teachers G, H, I and J) were fully transcribed. Selected interview excerpts have been translated from Hindi into English in this paper.

Themes in this paper were arrived at by data triangulation considering questionnaire responses, interview data, notes of discussions with teachers during the collaborative phase and video recordings of the workshop sessions. Correlation matrices were made for each part of the questionnaire. Then we reviewed the interview transcripts to identify responses related to teachers' views about mathematics and what they considered important in mathematics teaching and learning. The themes that emerged as important from the teachers' responses related to the following:

1. Primacy of obtaining correct solutions to problems through learnt procedures
2. Importance of repeated practice
3. Importance of teaching by showing procedures or solved examples
4. Dealing with student errors

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1 The number of male teachers at the primary level is much smaller than female teachers in this school system.

We used these themes to revisit teachers' responses to items in the questionnaire. Nine statements of belief taken from two parts of the questionnaire focusing on teachers' beliefs about mathematics, and its teaching and learning were identified as important for the themes listed above. Table 1 shows the distribution of responses of the 13 teachers to these statements from “strongly agree” to “strongly disagree”. Table 2 shows the correlation between the responses to the statements.

No.	Statement	Strongly agree	agree	Unsure	disagree	Strongly disagree
1	Students should be allowed to make mistakes and then discuss them.	3	9	0	1	0
2	Being good at mathematics means being able to perform calculation quickly and accurately.	1	4	2	4	2
3	If a student practices solving all the problems in the textbook two or three times, that is the best way to learn mathematics.	1	2	2	6	2
4	The key to learning mathematics well is to repeat the textbook exercises two or three times (or more).	0	4	0	6	3
5	The best way to teach mathematics is to explain one procedure at a time on the blackboard and then to make students practice it.	1	6	1	4	1
6	When students make errors, the best remedy is to make them repeatedly practice these types of problems.	4	4	2	1	2
7	Students cannot discover procedures (methods) for calculation on their own. They need to be taught these procedures. (There may be rare exceptions.)	1	7	0	3	2
8	A teacher should teach each topic from the beginning assuming that the students know nothing.	4	6	0	1	2
9	A teacher should explain things carefully in the beginning so that students can avoid mistakes.	3	8	0	2	0

Table 1. Frequency distribution of teachers' responses to selected items in questionnaire

	1	2	3	4	5	6	7	8	9
1	1.00	-0.76*	-0.23	-0.31	-0.56*	-0.57*	-0.58*	-0.59*	-0.68*
2	0.76*	1.00	0.49	0.21	0.66*	0.63*	0.55*	0.56*	0.74*
3	0.23	0.49	1	0.66*	0.74*	0.59*	0.72*	0.54*	0.62*
4	0.31*	0.21	0.66*	1.00	0.76*	0.50	0.74	0.56*	0.61*
5	0.56*	0.66*	0.74*	0.76*	1.00	0.80*	0.70*	0.70*	0.73*
6	0.57*	0.63*	0.59*	0.50	0.80*	1.00	0.51	0.37	0.51
7	0.58*	0.55*	0.72*	0.74*	0.70*	0.51	1.00	0.63*	0.72*
8	0.59*	0.56*	0.54*	0.56*	0.70*	0.37	0.63*	1.00	0.83*
9	0.68*	0.74*	0.62*	0.61*	0.73*	0.51	0.72*	0.83*	1.00

Table 2. Correlation matrix of 9 items (* values are significant at .05 level)

(Values from 1 to 5 with “Strongly agree” =1 and “Strongly disagree” =5)

Teacher Code	A	B	C	D	G	H	I	J
Teachers' mean on scale	2.7	4.4	2.2	2.3	3.9	2.9	2.9	2

Table 3. Teachers' mean scores on the scale (after reverse coding for Item No. 1 in Table 1)

Results

Table 1 shows that for most statements, there were differences among the teachers, with some agreeing and some disagreeing, indicating that the items are able to discriminate among teachers who have different articulated beliefs about teaching mathematics related to the themes listed above. The inter correlation (calculated taking values from 1 to 5 with "strongly agree"=1 and "strongly disagree"=5) among the items are significant for most of the items at .05 confidence level interval (above 0.51) and some for .01 (above 0.64) and .001 (above 0.76) levels. This suggests that the items potentially form a scale. The cronbach alpha of these 9 items is 0.91 (after reverse coding for the negatively correlated item no. 1) indicating good internal consistency. The means of teachers on the putative scale indicate the variability among the teachers responding to the questionnaire with the highest mean of Teacher B with mean 4.4 and lowest mean of Teacher J of 2.

In the interviews, teachers often expressed views that tended to agree with statements 1 to 9 in Table 1, that is, tended more towards the lower end of the scale than the distribution of responses in Table 1 suggests. The responses of Teacher G, in contrast, were generally consistent with her position closer to the upper end of the scale. We will discuss below some excerpts from the interviews along with the questionnaire responses under the four themes identified previously.

Primacy of obtaining correct solutions to problems through learnt procedures

Teachers responses in Table 1 show variation in agreement for viewing learning of mathematics as learning to obtain correct solutions while interview responses and classroom observations indicated that most teachers gave primacy to correct solutions. In the interview teachers were asked to describe a typical mathematics lesson that they taught. Most teachers described how they try to elicit the correct solution from students by explaining the question, sometimes focusing on textual cues and showing or recalling solutions to similar problems. The primary teachers focused on identifying the correct operation to be performed with numbers while middle school teachers focused on methods and specific types of questions and their solutions.

> First I explain one or two times then answer comes out orally, If not I solve the question... I explain on the blackboard and then give some connected problem. Sometimes I solve on the blackboard and then give them. (Teacher G)

Table 1 shows that 8 teachers agreed or strongly agreed that students cannot come up with solutions on their own, while 5 teachers disagreed or strongly disagreed with this view. Teachers G, H and B, who were among the latter, said in the interview that students could come up with the procedures of their own. But deeper probing revealed that they felt that students can come to know different procedures from elsewhere (magazines, parents) and only a few intelligent students can discover procedures and thus they have to be taught.

> Children bring their original ideas... their world is very big... he learns from lot of things other than teacher in the school e.g. Internet, father in specific profession.... Very few children able to give justification and explanation on their own. (Teacher G)

> Justification and explanation they can arrive [at] if the child is bright. (Teacher B)

The new textbooks in primary classes have less emphasis on standard algorithms for calculation than earlier textbooks, and greater emphasis on understanding problems situated in contexts and encouraging students to find their own ways of solving them. The discussion of problems in the classroom however still involved focus on the procedures and getting right answers to the problem. In most of the lessons observed the discussion of a problem typically ended with determining the correct answer and the student or teacher writing the correct solution or method on the blackboard and few opportunities for students to voice their understanding or develop strategies of their own. Students too expect discussion on right answers and method and even when a teacher tries to initiate a discussion of why the procedure works, students find it difficult to engage with it. In Teacher G's class most of the students went to private tuitions and often many already knew the correct answers when the teacher discussed a problem from the textbook. The teacher then called students to the blackboard to solve the problem and evaluate the method. When she tried to focus students' attention on understanding the concept, students did not engage readily as they already knew the procedures to get the correct answers. During the collaboration phase, her focus on asking why questions and developing justifications through visual representations however did succeed in engaging some students and in developing reasons for their answers.

Importance of repeated practice

The focus on procedures in teaching is consistent with giving students many similar problems for repeated practice assuming that it will help students in memorising the procedure as well as knowing which procedure is to be adapted for solving a certain kind of problem. Responses to questionnaire items related to this theme (Statements 3, 4, 5 and 6) show a variation, indicating that some teachers considered practice to be important while others did not. During the interview however, teachers affirmed the importance of practice. Some teachers felt that the textbook did not contain enough practice questions and so they themselves made practice questions for students. This was more common among the middle school teachers while primary teachers mostly asked students to repeat the work done in class as homework. However, Teacher A diverged from others as she felt that repeated practice of very similar problems amounts to rote memorisation.

> I don't repeat problems done in class. This practice is there in coaching classes which emphasizes rote learning. (Teacher G)

> Practice is required to keep what is learnt in memory so that they don't forget by the next chapter.(Teacher B, who had agreed with statements 4 and 5, but disagreed with statement 3)

> Homework is similar problems to those done in class. I give 50 problems in 2 days...Because in one day when they solve [many] problems then only they get it. (Teacher A, who had disagreed or strongly disagreed with statements 3, 4 and 5)

> They should become thorough means that they have to do lot of questions even when they know the principle. Along with that whatever is there in the textbook... I give similar questions for homework by changing the angles, distance, speed to make them thorough. I make students repeat the questions if they are important like linear equations, age problem, upstream downstream questions, because I want to make them familiar with the sentence language. I am doing on the blackboard and they are copying so whether they understood or not, I can know only when they do it again. (Teacher C, who had agreed or strongly agreed with statements 3, 5 and 6)

Importance of teaching by showing procedures or solved examples

In the questionnaire, the teachers responded to the following two statements.

1. The best way to teach mathematics is to clearly show the procedures (methods) to solve the mathematics problems.
2. The best way to teach mathematics is to show students how to solve some example problems.

There was considerable variation in the responses to these items and the correlation between them was zero. We believe that some teachers may have interpreted these items as mutually exclusive. The purpose of these items was to probe teachers' views on *showing* procedures or solved examples. So we combined responses to these items by taking the minimum of the response codes to the two items, which showed whether teachers agreed with at least one of the statements. The combined responses to the two items indicated that 9 teachers agreed or strongly agreed with at least one of the statements. 4 teachers disagreed or strongly disagreed with both the statements. The combined response of two statements was significantly correlated (at .05 confidence level or more) with all statements in table 1 except statement 4.

In the interview while talking about a typical lesson teachers described how showing solutions and steps of procedure is a major routine in their classroom teaching. Teacher G and A had consistently disagreed with the two statements, yet their explanations were focused on procedures.

> Explanation is the explanation of steps of procedure... (Teacher A)

> I tell them the topic, I generally do 2 questions in the class. One I explain properly...second one is done by students... (Teacher D agreed to 1, unsure about 2)

> After giving explanation for 2-3 examples, page number, activity is there then you can solve the questions. (Teacher H disagreed to 1 and agreed to 2)

Teacher H also elaborated about the lesson that she liked in the new textbook where lots of questions for different operations were there. She said that she made lots of similar questions for students. She felt “that is maths, multiplication and addition is coming [i.e., present]”.

Teacher B who had agreed to first statement and disagreed with the second one, elaborated how participation in the workshop and observing teaching focused on concepts has lead to her to re-evaluate the way she taught mathematics.

> After attending this course I feel it is possible. We used to teach everything in textbook. I saw yesterday the observation of classes. After that I felt that teacher can just give the concept and need not solve each and every question. Not giving the question but giving the concepts. If this method is adopted – give the concept and steps and all are secondary... (Teacher B)

Avoiding student errors

Although teachers reported change in their teaching approaches after the introduction of new textbooks, there was indication that not much had changed in terms of learning opportunities for students. One of the objective of the new textbooks was to allow children to "generate new knowledge" through imaginative activities and questions (NCERT, 2006, Foreword). However teachers' views were not conducive to meeting this objective. As we saw earlier, teachers did not really think that students could come up with solutions on their own. Moreover, their responses reflected a concern with channelling students' thinking in ways that would prevent them from making errors. 11 teachers either agreed or strongly agreed that teachers should explain things carefully in the beginning so that students can avoid mistakes while most teachers (9) also agreed that students be allowed to make mistakes and discuss them . This can be explained by teachers having acquiescence bias. Alternatively, it could be that teachers do not consider these two items as mutually exclusive in the sense that they teach in manner where they try to tell students clearly how to solve something but are aware that mistakes do happen and discuss mistakes. Classroom data corroborated that errors were avoided and when they occurred, students were told to how to do the problem correctly.

The interview responses and classroom observations also indicated that teachers rarely made efforts to understand why the student had made the mistake. The strategies that teachers adopted for addressing student errors was to indicate where the correction has to be done and asking students to do repeated practice (Table 1, statement 6). In the interview teachers said that they also ask 'good' students to explain to other students. However many felt that errors had to be rectified immediately .

> Wrong answer get corrected on the board only (Teacher J)
>
> If somebody gets wrong answer I ask him/her to do again or I tell that you have done steps till here correctly (Teacher G)
>
> If somebody got a wrong answer I help by first looking at their knowledge. If it is a knowledge problem I tell them what to do e.g. Formula..If they make calculational mistake I tell them. (Teacher D)
>
> Which step they have gone wrong I will tell. (Teacher C)

In one of the workshop sessions, teachers discussed student errors and the possible causes of such errors. After a long discussion on various hypotheses about what the underlying thinking of students that may have led to the errors, one of the teachers remarked, "In fact we know their [students'] mistakes but we don't really see into their thinking".

Discussion: Relation between beliefs and Knowledge

The findings discussed above suggest that teachers' beliefs that shape their practice may be closer to a transmission view of mathematics, that emphasises obtaining correct solutions, explicit teaching of solutions by showing, repeated practice, and avoiding or dealing with errors by emphasizing correct solutions. Teacher exposure to ideas of curriculum and textbooks could be the reason why some teachers agreed with constructivist view in the questionnaire but are unable to enact them in practice as suggested by interviews and classroom observations. This explains the inconsistent responses e.g. Item 1 and 9 in the questionnaire. Because of the social position of teachers as "curriculum implementers" it would be difficult for them to acknowledge views different from the ones prescribed by national curriculum. This further increases the complexity for ascertaining beliefs as professed beliefs might not be "true" beliefs held by teachers while teachers' practice might be a better reflection of the beliefs held by teachers. It also reflects that just by exposure to a different curriculum or textbooks it may not be possible to change teachers' "enacted beliefs".

The teachers' interview responses suggested one possible reason why these beliefs may be difficult to change, despite teachers' exposure to and awareness of the new ideas embodied in curriculum and textbook revision. In the questionnaire, teachers responded to the statement "It is important for students to not only know procedures (methods) for calculation but also why the procedures work." All 13 teachers either agreed or strongly agreed with this statement. However, the interview responses and interaction during the workshop showed that teachers themselves were unclear about why mathematical procedures work.

In the interview when primary teachers were asked how would they explain a division of say 36036 by 9, 3 out of 4 primary teachers explained the procedure. Only Teacher A said that she will focus on place value while explaining the procedure and might use the analogy of sharing in groups if a student has not understood. She asserted that “concept and procedure both should be emphasised”. Teachers often resisted engaging with conceptual explanation as they felt that students might get confused.

> Just like you have names these digits have names as ones, tens, etc....In addition we start from ones but for division we start from left to right...3 is less than 9 so you can't divide because 3 doesn't come in 9's table. So then you should take 2 numbers... small classes if you say about tens place, hundreds place, children will get confused... but you can say for addition and number names. We have to see that child is not confused and the child gets confident. (Teacher J)

Thus relationship between beliefs and practice might be more complex with knowledge about mathematics, its teaching and learning playing as an intervening variable in bringing about belief change. This calls for studies where teachers' development of knowledge is seen in relationship with the change in “enacted beliefs” that are held by teacher and exploring the process through which such knowledge can be developed.

The study also raises questions on the use of questionnaires to assess beliefs of teachers. The questionnaires afford assessment of large number of teachers but fail to capture the complexity of beliefs as to how different beliefs are connected with each other and the factors that might be influencing activation of some beliefs and thus influencing teaching. Agreeing to an item in a questionnaire does not rule out existence of other conditional factors or other connected beliefs that exist. For e.g., teachers agreed that students can come up with their own procedures but probing revealed that they were thinking only of a classroom situation where students might share procedure learnt from magazines, parents. For other items also where teachers agreed for more constructivist view in questionnaire, the data from interview and classroom observation was inconsistent with articulated beliefs.

The findings described here as well as findings from studies conducted elsewhere suggest that teachers' enacted beliefs are more resistant to change than their explicit assent to reform oriented views. Further, the findings suggest interaction between beliefs and knowledge, that inadequate mathematical knowledge may be a hurdle to change of belief. This may have implications for teacher development programs, where there is a need not only to create spaces for reflection on and revisiting of beliefs, but also for strengthening teachers' mathematical knowledge.

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