The measures for understanding teachers' mathematical knowledge for teaching fractions – how do they really work?

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Mathematical knowledge for teaching is the area researched by many researchers across countries. That effective and good teaching is dependent on teachers' own understanding of mathematics is well understood in the field. At HBCSE, with experience of several years of in-service teacher education the nature of teachers' knowledge is conceptualized as being composed of two major areas (teachers' content knowledge and learning pedagogic techniques), found similar to what others have proposed with some cultural variations. The question which I try to address here is what role does the assessment measures developed for understanding teachers' mathematical knowledge, play in teachers' cognition? Do the assessment measures themselves help learning of that particular topic? I discuss here the case of fractions and try to understand the learning that occurred during interviews and discussions with the teachers subsequent to their written response to the assessment tasks.

Introduction

Concern about students' learning of mathematics has directed the attention of everyone towards the teachers' understanding of mathematics. Observation of mathematics classes suggests that teachers' knowledge of mathematics and their ability to deploy it in teaching, matter for the quality of students' opportunities to learn (Ball, et al 2004). But still what constitutes 'knowledge of mathematics for teaching' is not commonly defined. The domain of

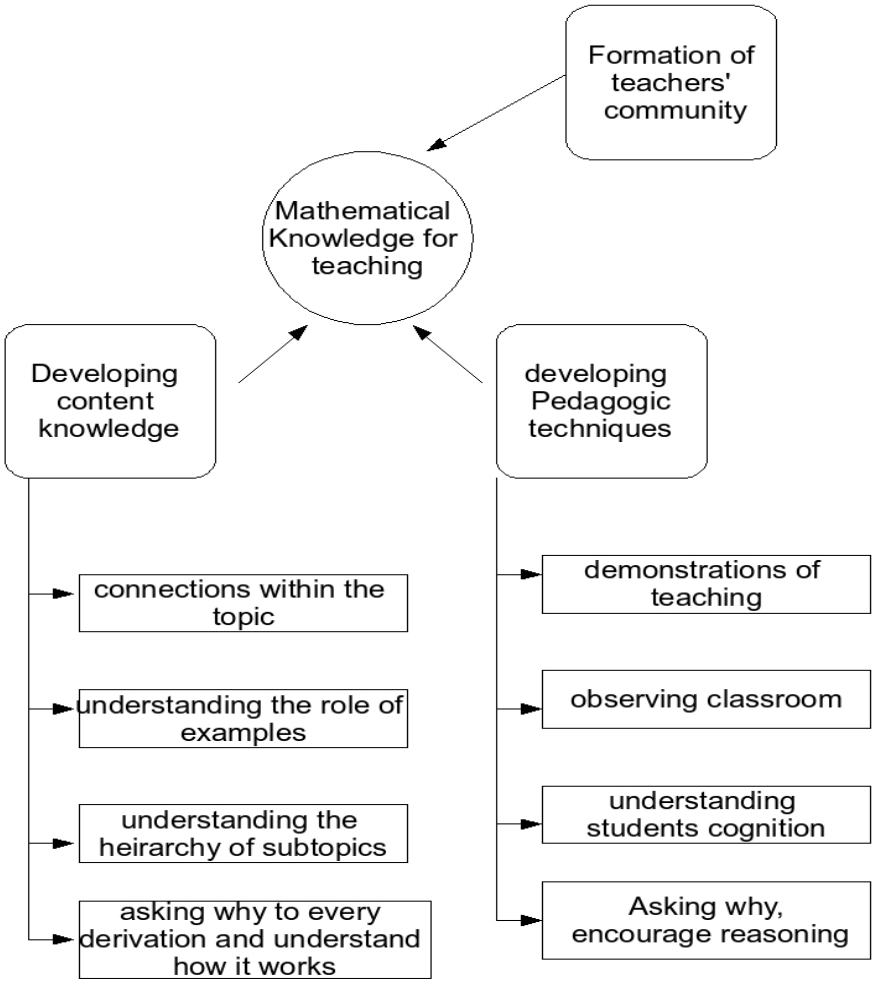
teachers' knowledge identified by Shulman (1987), which he termed as 'Pedagogic Content Knowledge', made the distinction between knowing the content for 'oneself' and knowing it with pedagogy required for teaching this content. This idea by Shulman focused teacher education on the content knowledge required for teaching. But if teachers' knowledge of content is inadequate, then it is not surprising as they are the products of the system which we are trying to improve. In India it is also true that no substantial inputs are available for teachers in their pre-service teacher education courses to develop the mathematical understanding required for their own teaching (Naik, 2008; NCTE, 2006). It has also been argued that throughout the past twenty years, researchers have used the term "pedagogical content knowledge" to refer to a wide range of aspects of subject matter knowledge and the teaching of subject matter, but still the potential of the term remains insufficiently exploited (Ball et al, 2007).

According to this view, mathematical knowledge for teaching goes beyond that captured in measures of mathematics courses taken or basic mathematical skills. For example, teachers are not expected to only calculate correctly but also to be able to justify each and every derivation with possible representation. How is this knowledge attained? As Ma (1999) describes Profound Understanding of Fundamental Mathematics (PUFM) is attained in Chinese teachers in their pre-teaching courses and in actual teaching careers by following means -studying teaching material intensively, learning Mathematics from colleagues, learning Mathematics from students and learning Mathematics by doing it. So this gives us insight that, the knowledge of mathematics which is tailored to the work teacher do with curriculum materials, instruction and students is attained by doing activities pertaining to the profession of teaching. In HBCSE, we[1] have also developed some measures for checking this knowledge of mathematics. I am trying to make an attempt to see teachers thought processes in attempting such mathematical tasks meant for checking their mathematical knowledge for teaching.

Theoretical framework of teacher workshops at HBCSE

The experience of in-service teacher education at HBCSE has led us to conceptualize teachers' knowledge as being composed of two major domains content knowledge and knowledge of pedagogic techniques (See fig. below). Here the term 'Pedagogic techniques', is to explain the association between content knowledge and the instruction/demonstration.

The workshops talk about the techniques which are non-subjective that is independent of the individual teacher but particular to the subject of mathematics. For example, pattern development and generalisations. The other attributes of the content of the workshop will be discussed in the paper.



Domains of Teachers' knowledge

Measures for understanding Fractions

Teacher education requires some care which we take during adult education. Hence it puts limits on the way questions can be asked to the teachers about their own understanding of the content. We ask questions in the format which they are very familiar with. The question

paper for teachers on mathematical knowledge for teaching includes four or five solutions to each of the question given (as these are the responses given by different students) and they need to check each of the solution whether it is right or wrong. An example of such question is as follows-

> *In the following question answers given by students are given as options. Check each option whether it is right or wrong.*
>
> *7 2/5 – 7 × 2/5 = _____*
>
> *a) 0 b) 2/5 c) 4 3/5 d) 23/5*

The question with multiple responses challenges the teachers' knowledge and beliefs about the concepts. The conversions of mixed number into fractions are learned as multiplication of the whole number and the denominator followed by addition of the numerator. This procedural understanding develops a belief of the existence of a multiplication sign between 7 and 2/5. The existence is also supported by the rules from algebra as it is often said that if there is no sign between two letters (or a letter and a number) then there is a multiplication sign. So *xy indicates that x × y. Similarly,* 7 2/5 indicates 7 × 2/5. These lead teachers towards wrong answer i. e. option (a) 0. But interestingly when these teachers are interviewed and asked about the reasons for the rest of the answers, they started thinking about students reasoning. This unpacking of what students thought, gave them insight about the structure of fraction notation itself.

The interview with teachers showed that they know how to carry out the multiplication of fractions or fractions with the whole numbers (numerator × numerator/ denominator ×denominator*)*. They also know procedurally how to convert a mixed number to the fraction form. But the task above presented them with the situation of conflict. This conflict created the need to understand the relationship between procedure and meaning of it. The task gave them the platform to change their representation of fractions. For example, one teacher who had earlier made the error of equating 7 2/5 – 7 × 2/5 to zero, argued as follows -

7 2/5 = (7 ×5 + 2) / 5

= 37/5

= 35/5 + 2/5

= 7 + 2/5 = 7 2/5

The derivation above was a rediscovery for that teacher as she proved that there is no multiplication sign in 7 2/5 but 7 and 2/5 has operation of addition in between them. To obtain a response of this kind, a pre-requisite is that the teachers should understand fractions. This in-service teacher who already possess some knowledge of fraction and have taught to students could correct her own understanding of the concept with the inputs obtained from the assessment measure and the conflict aroused because of it. Such opportunities may not be available for teachers in the traditional textbook assessment questions. Also re-teaching the concept of fraction to teachers might not create any challenges to the existing knowledge of theirs. But an example such as above gives them the platform to challenge their own understanding, repair it and reform it.

I worked with three different groups of teachers. The data is collected in the form of their written responses and few selected interviews. The analysis of the data will be presented in the extended version of the paper. The written task includes few more topics of elementary education like multiplication and division, but interviews are taken only for selected items of fractions. The method and analysis indicated that such tasks are potential for teacher education and gives us a tool which can be used for Teachers' content development.

Note:

1. I acknowledge the role of Dr. K. Subramaniam, HBCSE who had major contributions in the development of these assessment measures.

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