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A WAY FORWARD FOR TEACHING IN MULTILINGUAL CONTEXTS: PURPOSEFULLY RELATING MULTI LINGUAL REGISTERS

Susanne Prediger¹, <u>Philip Clarkson²</u>, Arindam Bose³

¹TU Dortmund University, Dortmund, Germany, prediger@math.uni-dortmund.de

² Australian Catholic University, Melbourne, Australia, Philip.Clarkson@acu.edu.au

³ Homi Bhabha Centre for Science Education, TIFR, Mumbai, India, arindam@hbcse.tifr.res.in

Abstract. In this paper we first revisit three traditions of dealing with linguistic transitions between registers and representations. We then suggest an integrated approach of purposefully relating registers. This new approach will enhance language-sensitive teaching strategies in multilingual classrooms that aim for conceptual understanding.

Key Words: Multilingual Mathematics Classrooms – Teaching Strategy – Relating Registers

In the literature, there are three different strong ideas related to different language registers and discourses that have not been closely linked. Indeed they are often treated as quite distinct entities. These are code switching, transitions between informal and academic (mathematical) forms of language within a given language, and transitions between different mathematical representations. By exploring the overlap between these three ideas and in particular by articulating their interconnections, new insights and implications are gained. Rather than three apparently discrete sets of ideas, we will present one integrated set of ideas for teachers and researchers, which in turn has the potential to drive theory, curriculum and teaching developments.

The purpose of this paper then is two-fold: to build theory and then suggest teaching strategies that could be used with multilingual mathematics learners. The teaching strategies are based on the *approach of purposefully relating multi lingual registers*, an approach that integrates the three research traditions and classical teaching strategies.

THREE TRADITIONS OF REFLECTING ON LINGUISTIC TRANSITIONS

Transitions between first and second languages: Code-Switching

The connection between first and second languages in mathematics classrooms has been mainly researched and discussed in terms of code-switching that occurs quite naturally for people who use two or more languages. Code-Switching refers to the practice of switching between two or more languages in a conversation or an utterance (Farrugia, 2009). Sometimes code-switching is prompted by specific contexts, and often such switching or mixing of several words of one language into the utterance of another happens quite unconsciously in the flow of conversation between conversant multi linguals.

Learners' and teachers' mathematics classroom activities that involve first and second languages have been analysed very carefully in terms of code-switching and their variants, conditions and benefits for mathematics learning (Barwell, 2005; Bose & Choudhury, 2010; Clarkson, 2007; Cummins, 2000; Faruggia, 2009; Halai, 2009; Setati, 2005). Many of these descriptive empirical studies have proved to be insightful, especially in relation to cognitive, political and social issues. For example, the 'threshold hypothesis' of Cummins (2000) says that students are at cognitive advantage with high proficiency in two or more languages and neither at advantage nor at disadvantage with proficiency in only one language (Barwell, 2009). For our purposes, it has been shown that proficiency in two languages enhances meta-cognitive skills in mathematics among students (Clarkson, 2007). Moreover, at a time of global mobility when immigration and family relocation is ever increasing, multilingualism has become ubiquitous, overt and tacit (*ibid*). Here emerges the 'political dimension of language use' as in the case of South Africa (Setati, 2005) or acknowledging the positive effects of language transitions in the curricular documents in multilingual societies like that of India and Pakistan (Bose & Choudhury, 2010; Halai, 2009).

What has been often emphasized in these studies are the benefits of code-switching, particularly when vivid transitions between languages occurs, since these moments allow simultaneous learning of both language as well as mathematics. Additionally, code-switching can be used as a mark of solidarity empowering the students in the classroom. To sum up, these empirical analyses show that code-switching does provide comfortable and flexible modes of communication, and is thus clearly a useful pedagogical resource for mathematics teaching.

Building upon this empirical and political work, it is time not only to *allow* spontaneous code-switching and descriptively analyze its conditions and effects, but to develop and promote teaching strategies that make more purposefully use of code-switching and other links between first and second languages. In particular, teaching strategies are needed that will help teachers guide students to make conscious choices between their language registers as a possible solution strategy for co-learning mathematics. An aspect of the model will be to privilege students' competence in all their language registers.

Transitions between informal and formal language registers

Turning now to the second key idea; we first review some of the literature dealing with students moving between informal and formal registers within their dominant language, but we then extend this notion to multi lingual learners.

Within mathematics education research literature, Pimm (1987), Freudenthal (1991) and many others have advocated a careful transition from everyday language to the more technical language of mathematics as an important teaching strategy that enhances conceptual understanding of mathematical concepts and ideas. Empirical studies (e.g. van den Heuvel-Panhuizen, 2003) show the power of this teaching strategy.

However, some students seem to experience serious difficulties in making such a transition and it appears that the source of the problem is connected to an intermediate register between

the informal everyday register and the formal technical language. For a long time this intermediate register has been underestimated. For a theoretical explanation of this issue it is useful to use Cummins' distinction between BICS and CALP. Cummins (2000) has suggested that there is a distinction between surface fluency in an everyday language register and the language skills needed in a context of high cognitive academic demands. He developed the construct 'Basic Interpersonal Communicative Skills' (BICS) to describe the situation when there are contextual supports for language. Face to face conversations, for example, provide actions with hands and eyes, instant feedback, and other cues to support meaning. Such situations are said to be "context embedded" (Koch & Oesterreicher, 1985) and surface language skills are sufficient. On the other hand, where higher order thinking skills such as analysis and evaluation are required (for example in problem solving contexts), language becomes "disembedded" from a meaningful supportive context and becomes more abstract. Such a situation can be thought of as "context reduced" (Koch & Oesterreicher, 1985; Schleppegrell, 2004) and need more explicit linguistic means to be mastered. Skills required to become fluent in this style of language fall in the domain of 'Cognitive Academic Language Proficiency' (CALP). The corresponding language register is here termed school register. By the notion 'school register', we refer to the term language of schooling as explored by Schleppegrell (2004) and discussed in many political contexts as in the European Council (Thürmann, Vollmer & Pieper, 2010).

Although many overlaps exist, we can for analytical reasons distinguish *school register* from *everyday register* (in which Pimm's informal language and Cummins' BICS is located) and *technical register* (which comprises mathematical technical language of school mathematics), hence giving a three-tiered model. Most teachers are aware that the technical register needs to be acquired in school, whereas the school register (to which students of privileged socio-economic background are often already acquainted) is sometimes treated as learning *condition* instead of as learning *goal*. In these cases, students with weaker language background, either because they come from a lower socio economic background or they are from a migrant community not speaking the language of schooling or both, experience difficulties.

Hence for language acquisition (of the *everyday register*, of the *school register* as well as of the *technical register*), it turns out to be important not only to transit once from the everyday register via the school register to the technical register, but to move flexibly forward and backward between all the three, as emphasized by Freudenthal (1991) and elaborated by Clarkson (2009). While extending the model for multi lingual learners, Clarkson (2009) adds the important perspective that the three registers might exist in more than one language for multi lingual learners (see Figure 1).

For mathematics classrooms, it is hence important to facilitate transitions between all these registers. It also becomes important for teachers to understand the possibilities that are afforded to students to gain deeper understanding if they are encouraged to use their languages effectively. Some monolingual teachers, with a mindset that suggests that mathematics learning is somehow unrelated to language use, have been surprised to realise

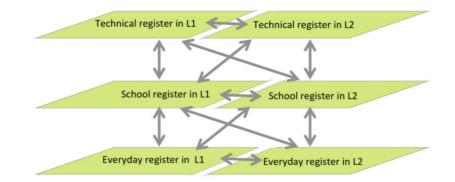


Figure 1. Enlarged model (adapted from Clarkson 2009)

the use multi lingual students make of their languages in all learning situations, including mathematics, particularly in the everyday register (Clarkson, 2007). But the contrary can also happen: When some teachers have tried to use students' everyday language to set the context for deeper learning, there has been a misunderstanding when transitioning between the L1 and L2 everyday registers, confusion rather than deeper learning can be the outcome (Halai, 2004). These very short examples give a first hint why it might be useful to care for the transitions more purposefully than it is useful in most classrooms.

Transitions between different mathematical representations

The third strong idea has been developed for all students, not for those with specific language backgrounds. Bruner (1967), Dienes (1969), Duval (2006) and many others have pointed out that understanding the relationships between different mathematical representations is an important activity for developing students' conceptual understanding. Many examples from the research literature indicate how this is employed as a fruitful teaching strategy, e.g. in task designs (Swan 2002). Duval (2006) in particular has given a semiotically grounded theoretical foundation why transitions between different modes of mathematical representations are crucial for the acquisition of conceptual understanding. He emphasized that the abstract nature of mathematical concepts is one reason for these necessities.

Although from the early days, language was seen to be an integral part of the mathematical semiotic registers, often the role of language and the manner in which teacher and students interacted was left unexamined. In particular the "linguistic" registers are not clearly differentiated: for example, students' preferences for using their L1 versus L2, and possibly occurring phenomena of code switching are not reflected in the model. Nor are the informal versus technical/formal aspects of language included in their models to any extent. We also note that there are differences in the way verbal and written forms of a language are expressed, but affordances or disadvantages of one versus the other are not investigated to any extent. These differences need to be acknowledged as another variation within the communications between classroom participants.

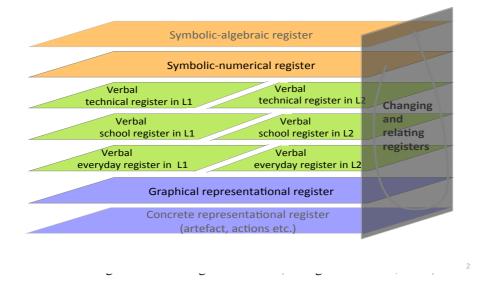
A common example of a good teaching strategy for primary children that has been in the literature for many years is that of 'think boards' (Haylock, 1984). A crucial aspect of this approach is the explicit use of language. In some versions both spoken and written language is

used, whilst in other versions only one is emphasized. However there is little explicit discussion of language in this strategy; it is just another way of dealing with the conceptual idea or the procedure as an alternative to using materials or visual cues or contexts. Absolutely no thought is given to the possibilities of using L1 or / and L2. Below we suggest that this deficiency needs to be addressed.

INTEGRATING THREE TRANSITIONS

A beginning of integrating the above three transitions

In Prediger and Wessel (2011), an integration of the above three traditions was suggested as the "relating register approach" (Figure 2). They present empirical evidence and explore some of the practical implementations to show the potential of this more complex and encompModel for integra ng registers (= languages and representa ons)



Within the last three years, this integrated model has proved to be very useful as a heuristic tool to guide the practical design and support of learning processes for multilingual learners (Prediger & Wessel, 2012), as well as empirical investigation (Prediger & Wessel, 2011). In particular the graphical or pictorial representation register turns out to be very useful for learners with restricted language capacities in L2. This register allows students to use deictic means (like "this" and "there") and contextualized language for expressing mathematical thoughts, so they can temporarily disburden the students. After having formed adequate mathematical ideas, they can concentrate on developing the language. Activities like collecting expressions on posters can focus the attention on language issues. For exemplification, Table 1 shows a collection of possible activities for relating registers (most in Prediger & Wessel, 2012, with some from Swan, 2002).

Table 1. Repertory of different activities for relating registers (Prediger & Wessel, 2012)

Activity	Examples for fractions
Translate from one register into another (freely chosen or determined)	 Here is a multiplication of fractions, find a situation for it or draw a picture. Here is a word problem in Turkish, translate it into German. Here is a quite complicated text of a newspaper. Write down a text that is easier to understand for your friends. (School language -> everyday language)
Find fitting registers, also for consolidating vocabulary	 On these 15 cards, you find fractions, situations and drawings, assign those that belong together. Add missing cards. Which of these words denote the same: denominator, deminator, rate, ration, part, whole. Write them together.
Examine or correct if different registers fit	 Tim has drawn this false picture for 4/5. Modify it so that it fits. The writer of the journal wanted to explain by a story, why 3/4 > 3/5. Explain his misunderstanding.
Explain how to find a mathe- matical relation or structure in a certain register	• How can you see in the picture that $3/4 > 3/10$?
Collect and reflect different means of expression within one register	 Collect different pictures for 2/5 x 3/4. Read the German and the Turkish newspaper and collect all ways how rates and rations are expressed (e.g. every third boy). Write them on a poster for the classroom.

Theoretical exploration: registers, languages, discourses or representations

In the integrated model in Figure 2, the different registers and representations were ordered hierarchically according to their increasing degree of abstractness (as proposed by von Kügelgen, 1994). However, the degrees of abstractness depends on how abstract or concrete the topics are, and whether or not the different levels are of the same quality. For a deeper conceptualization, different authors have suggested different theoretical constructs.

The linguist von Kügelgen (1994, p. 34) offered the construct 'concept levels', but this does not appear to be a good fit to our context because of a too strong hierarchy and too narrow a focus on isolated words, and hence does not reflect the complexity of language. The psychologist Bruner (1967) suggested the classical conceptualization of different 'modes of representation' (enactive, iconic, symbolic) which proved to be useful for designing learning sequences. But his use of the construct 'representation' lacks important dimensions such as references to contexts, functions, and social embeddedness. Hence the construct 'representation' might be understood to mean that there are one-to-one-translations between all representations, without shifts in meaning and function, which is definitely not the case in the scenarios we have built up in this paper.

The language education researcher Hallet (2012) focuses on 'symbolic languages' and emphasizes their different semiotic functions to describe and explain the world in specific ways. In mathematics education research, there is a focus on different semiotic functions in his construct of 'semiotic registers'. Duval (2006) for example, emphasizes that the meaning

("content") of a mathematical object can change with a shift of representation: "The content of a representation depends more on the register of the representation than on the object represented. That is the reason why passing from one register to another changes not only the means of treatment, but also the properties that can be made explicit" (p. 111).

With this observation, Duval notes an aspect that is equally important for the sociolinguistic construct of 'register'. Halliday (1978), a sociolinguist, defines register as a "set of meanings, the configuration of semantic patterns, that are typically drawn upon under the specific conditions, along with the words and structures that are used in the realization of these meanings" (p. 23). He distinguishes registers from dialects by defining dialects as ways to say the same things differently (which in a certain way also applies to representations), whereas he describes registers as "ways of saying different things" (p. 35). As a consequence, a change of register also implies a shift in meanings, or at least nuancing the meaning so some relationship, etc. may be more easily seen. Additionally, Halliday emphasizes the social embeddedness of the communication situation for characterizing registers: "A register can be defined as the configuration of semantic resources that a member of a culture typically associates with the situation type. It is the meaning potential that is accessible [only] in a given social context" (p. 111). Hence, registers are characterized by the types of communication situations, their fields of language use, the discourse styles and the modes of discourse.

The *everyday register*, *school register* and *technical register* can thus be characterized as registers in Halliday's sense, since as we have shown earlier these are used in different communication situations. But we would also stress although the core of each register is clearly distinguishable, the boundaries between them are not hard and rigid, but at times quite fluid. There is clear overlap of the contexts, especially between the technical register and school register; or, how could it be otherwise? We would also note that with the emphasis on authentic and / or real problems, teachers often deliberately seek to have students' transition between their everyday language register in describing and understanding such 'real' problems before reconceptualising it in the 'technical register. We also note that graphical and symbolic representations can be conceptualized as registers (Figure 2), but in Duval's sense. For Halliday's conceptualization, we would rather subsume the representations to different registers.

CONCLUSION AND OUTLOOK: TOWARDS A TEACHING STRATEGY

In this paper we have brought together three key ideas that have often been regarded as different, and have shown that their integration can offer a new way forward, theoretically as well as in terms of curriculum development and exploring more rounded teacher strategies. Nevertheless this beginning brings with it a range of questions, both, theoretical and practical. We therefore conclude by asking a number of these questions which may prove to be crucial for an ongoing research agenda:

• Do students who are taught using this model develop better understanding of the subject matter and /or perform better than other students?

- What are the similarities and differences in language processes / strategies that students use while moving between L1 and L2 as compared to moving up the tiered model?
- Which teaching strategies can possibly guide/ encourage students to use multiple languages and language forms?
- Are there central teaching principles at the core of different teaching strategies, with the strategies changing in response to the differing language contexts found in different countries and classrooms?
- Are there effective teaching strategies already in the literature that can be adapted to meet the conditions of this model?
- In particular, can open questions, authentic questions, and an adapted use of think boards, be used as teaching strategies that are useful / effective in implementing this model?
- Are strategies aimed at making 'language' more explicit in mathematics classrooms, for example, using displays such as 'word walls', helpful in focusing students' attention on language issues?
- What changes to mathematics curricular documents, beyond including vocabulary lists and glossaries, will make issues such as language and the way it is taught and used for both mono and multi lingual students one of the core components around which content and procedures coalesce, rather than the present structure that has content as the central component of such documents?

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