An invitation to

Critical mathematics education

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Concerns

- Critical mathematics education can be characterised in terms of concerns.
- It is not defined, for instance, through an educational methodology.

The concerns have to do with:

- Socio-economic diversity.
- (Lack of) equality/equity.
- (Lack of) social justice.
- (Lack of) students' autonomy.
- (Lack of) teachers' autonomy.
- Socio-economic functions of mathematics education.
- Socio-economic functions of mathematics.

Some concepts

Several concepts are important for formulating concerns of a critical mathematics education. For instance:

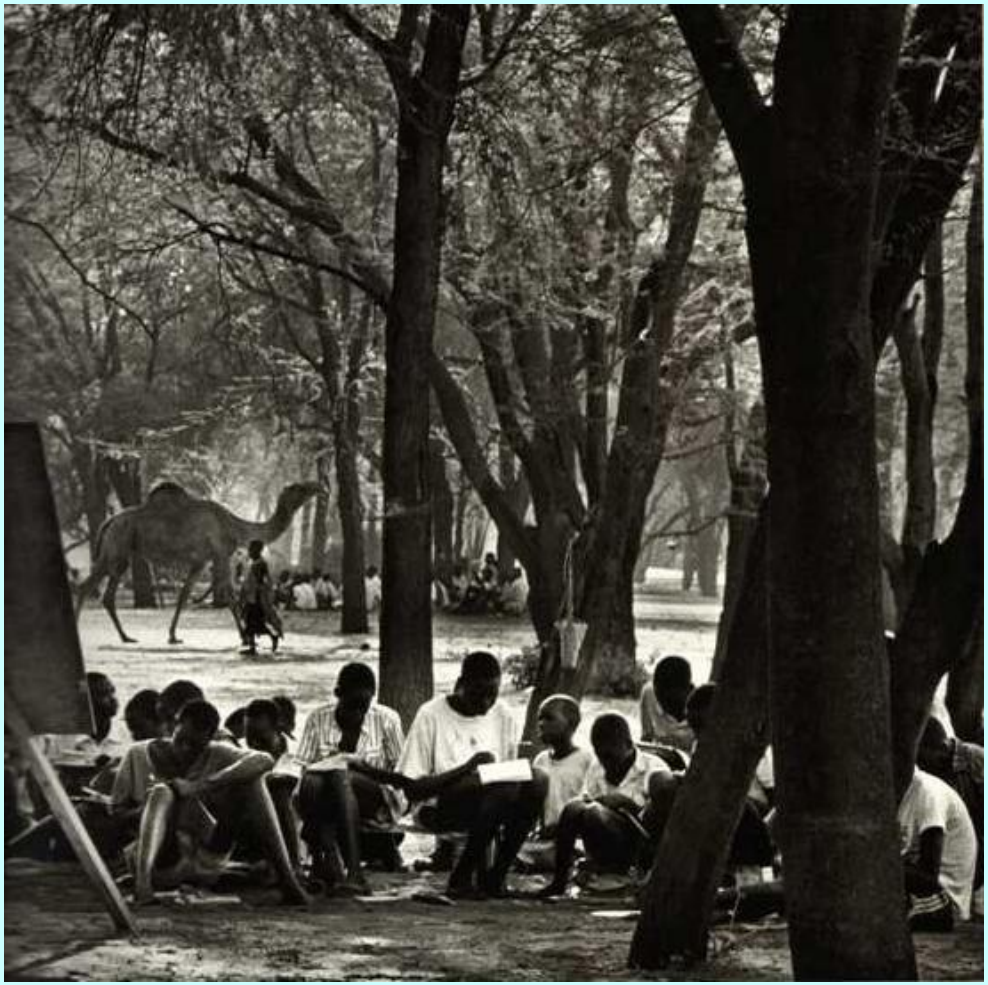
(1) *Situation (or context)*
(2) *Foreground*
(3) *Meaning*
(4) *Mathematics in action*
(5) *Uncertainty and doubt*
(6) *Critique*

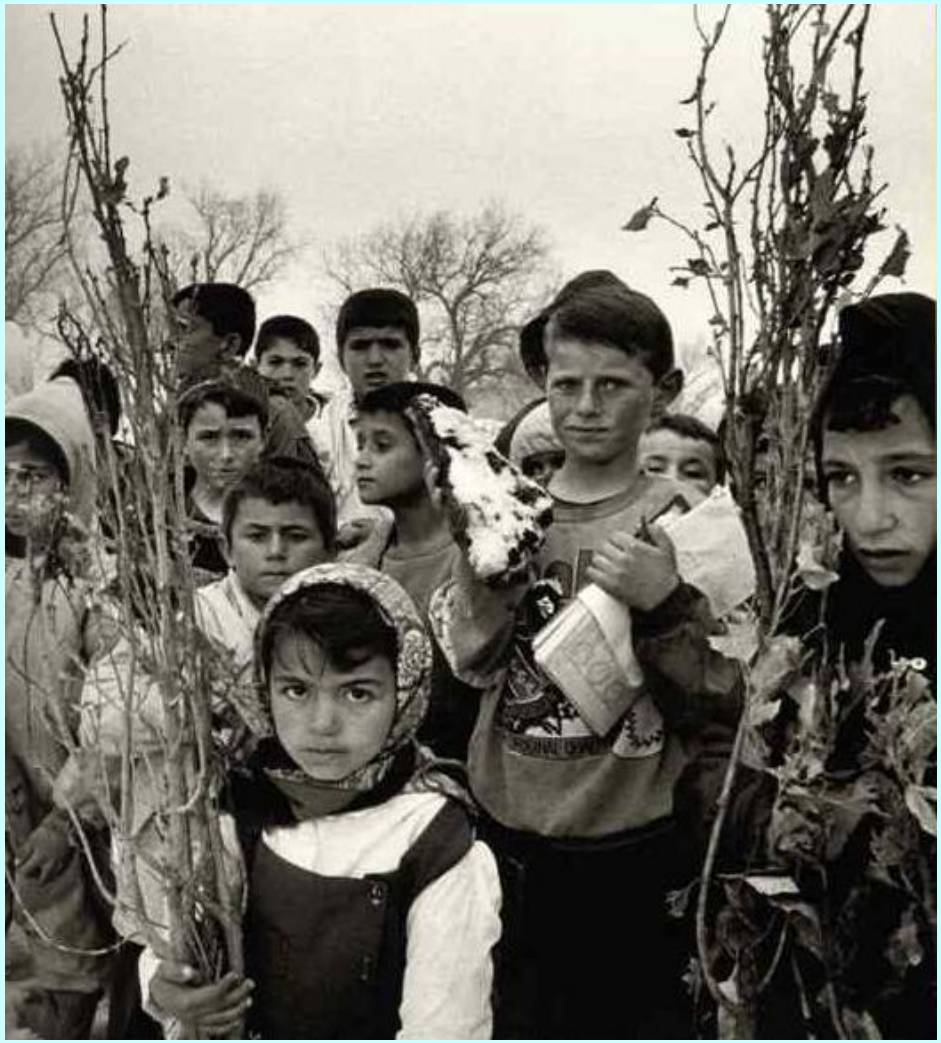
Other important concepts are: Power, dialogue, mathemacy, globalisation, ghettoising, landscapes of investigation, and pedagogical imagination.

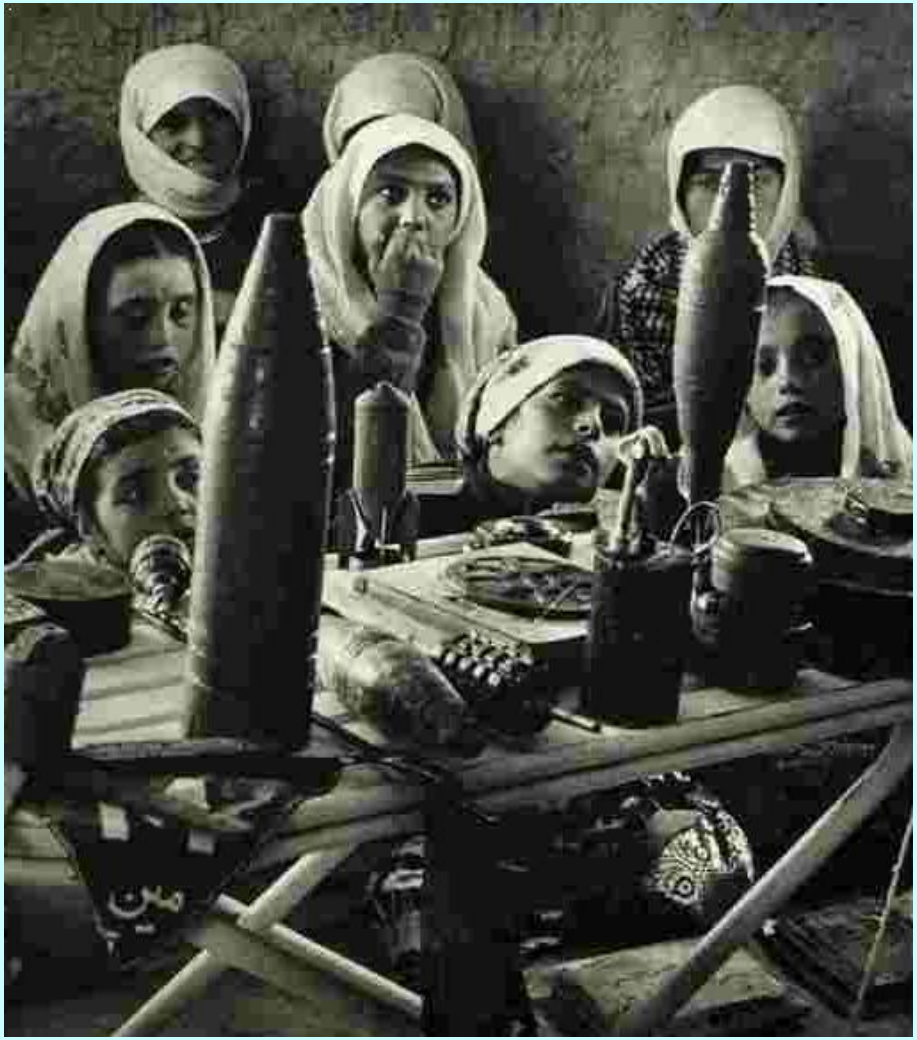
O berço da desigualdade
(The Cradle of inequality)

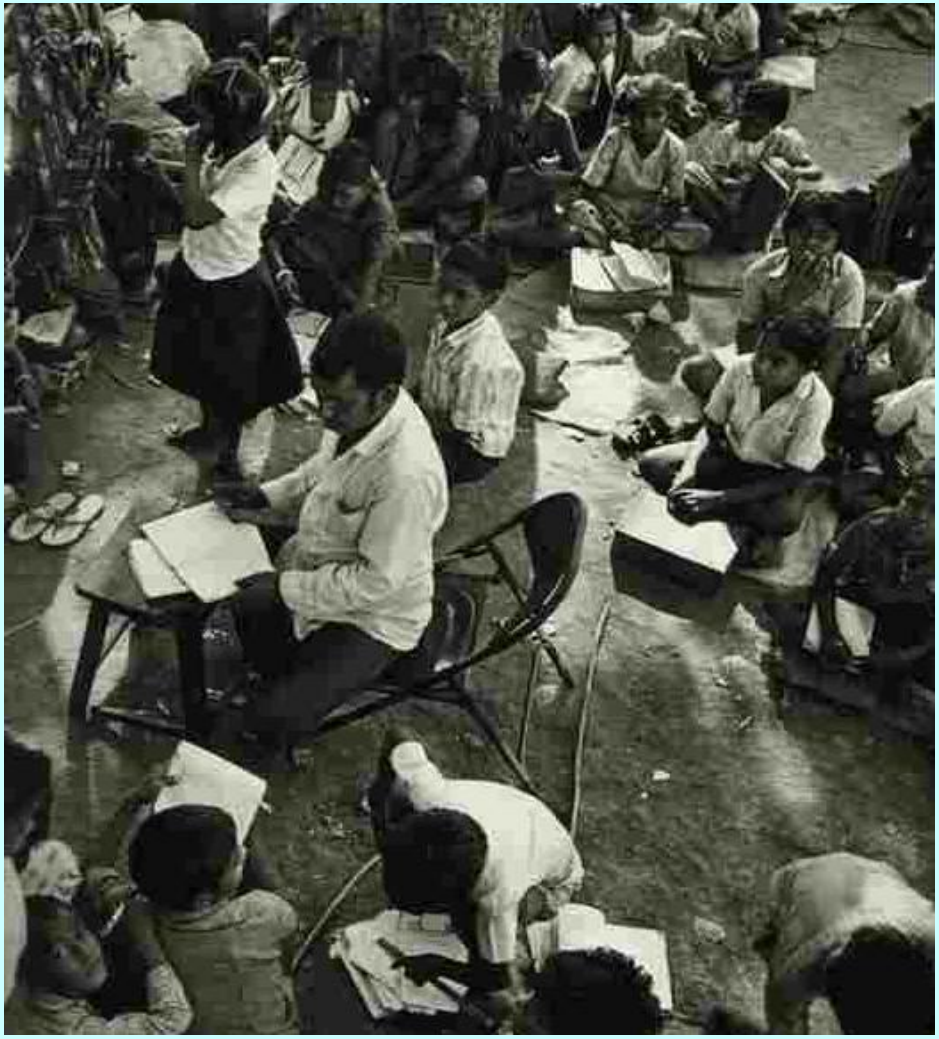
Photos by Sebastião Salgado
Text by Cristovam Buarque

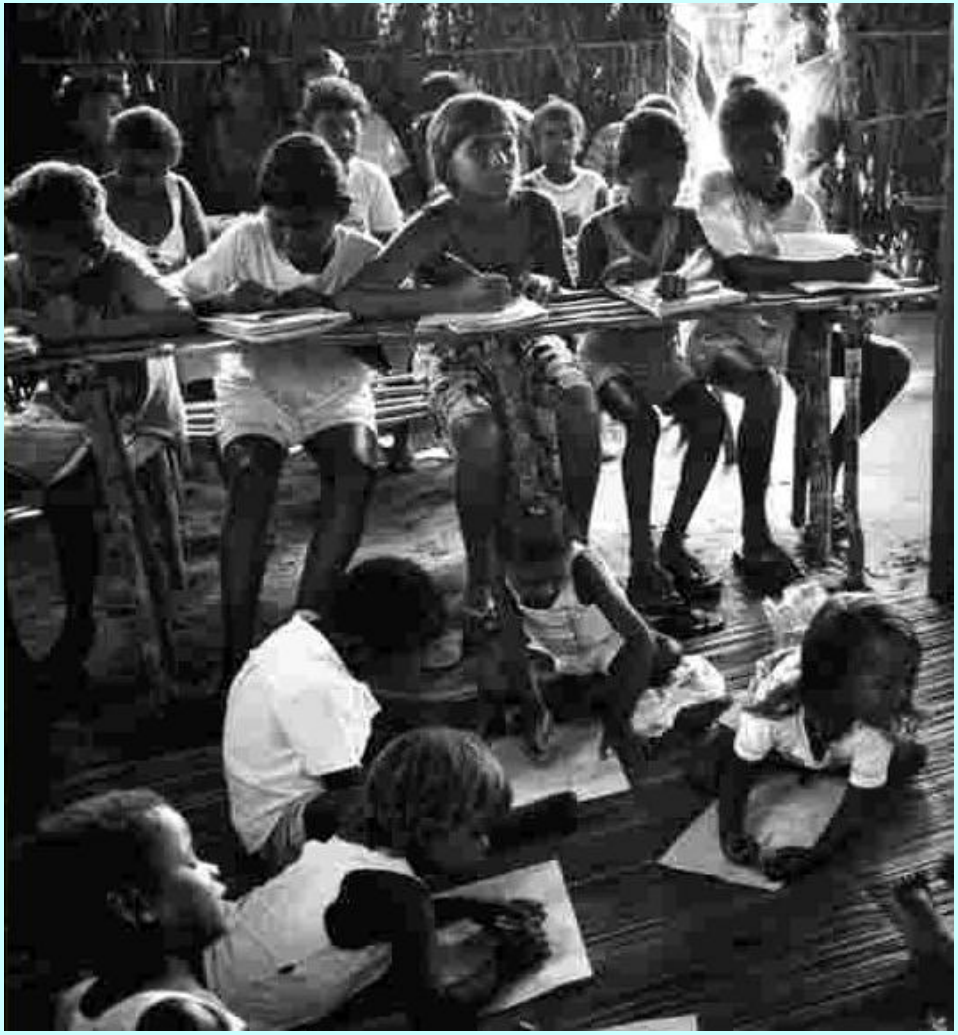
Editora UNESCO no Brasil











Some statistics

According to tradition it has been common to divide the world in three regions:

(1) So called more developed countries: Western Europe, USA, Canada, Japan, Australia, New Zeeland.

(2) So called less developed countries: Africa, Latin America, Caribbean, Most part of Asia, The Pacific, Arabic countries.

(3) So called countries in transition: Central Asia, Central and Eastern Europe.

Statistics

The world's population of children (between 6 and 11 years) are distributed in the following way:

(1) So called more developed countries: 10%.
(2) So called less developed countries: 86%.
(3) So called countries in transition: 4%.

In total 16% of the world's population of children does not go to school.

UNESCO (2000). *Education for All: Statistical Assessment 2000*. Paris: UNESCO,
http://unesdoc.unesco.org/images/0012/001204/1204/120472e.pdf

Stereotypes?

Classroom descriptions, normally presented in research journals in mathematics education, reveal a certain perspective on the domain in question.

These descriptions do not include much noise. The students have the necessary textbooks. They work at a computer, if necessary. The students are not hungry. There is no violence threatening the students. Etc., etc., etc.

The environment is pleasant (and stereotypical).

A paradigmatic prototype

- Maybe research in mathematics education has developed a rather particular perspective on the domain in question.
- Maybe mathematics education research is dominated by a paradigmatic prototype (stereotype).
- Theories in mathematics education – theories about learning, meaning, errors, achievements, etc – might be biased, due dominant paradigmatic assumptions.

(1) Situation

- For critical mathematics education it is important not just to focus on prototypical situations.
- It is important to develop ideas, notions and theories with references to all different teaching-learning situations.
- It is, for instance, important to address the learning situations of students from favelas in São Paulo, Paris, New York, Tokyo, etc.
- It is important to address the different learning situations shown in the photos from *The Cradle of Inequality*.

Different situations

- Maybe the photos from *The Cradle on Inequality* are special, only when we consider the majority of the developed theories of teaching and learning mathematics.
- But not any particular, if we consider the situations of the majority of children in this world.

(2) Foreground

By the *foreground* of a person I understand the possibilities which the social, political, economic, cultural situation (the socio-economic situation) makes available for the persons.

However, not the opportunities as they may exist in any objective form, but as they become experienced by the person.

A *foreground* expresses expectations, hopes, frustrations, uncertainties.

An illustration from…

World Bank (2006). *Equity and Development: World Development Report 2006*.Washington and New York: A co-publication of The World Bank and Oxford University Press.

Nthabiseng and Peiter…

"Consider two South African children born on the same day in 2000. Nthabiseng is black, born in a poor family in a rural area in the Eastern Cape province, about 700 kilometres from Cape Town. Her mother had no formal schooling. Pieter is white, born in a wealthy family in Cape Town. His mother completed a college education at the nearby prestigious Stellenbosch University."

Nthabiseng and Peiter…

"Nthabiseng has 7.2 percent change of dying in the first year of her life, more than twice Pieter's 3 percent. Pieter can look forward to 68 years of life, Nthabiseng to 50. Pieter can expect to complete 12 years of formal schooling, Nthabiseng less than 1 year. Nthabiseng is likely to be considerably poorer than Pieter throughout her life. Growing up, she is less likely to have access to clean water and sanitations, or to good schools…"

Parameters

- A foreground is structured through a range of parameters. These parameters signify tendencies (propensities). Some tendencies as strong (almost deterministic); some tendencies are weak.
- A foreground includes contingencies as well. Some possibilities seems to occur randomly.

A ruined foreground?

- A foreground could be ruined. (This does not mean that there is no foreground, but that it appears not to include attractive possibilities.)
- The foreground of groups of children could be ruined.
- There are many groups of Nthabisengs around the world.
- Let us take a new look at *The Cradle of inequality*.

A couple of references

- Skovsmose, O., Alrø, H. and Valero, P. in collaboration with Silvério, A. P. and Scandiuzzi, P. P. (2007). "Before you divide you have to add": Inter-viewing Indian students' foregrounds. In B. Sriraman (Ed.), *International Perspectives on Social Justice in Mathematics Education*. *The Montana Mathematics Enthusiast*, Monograph 1, 151-167.
- Skovsmose, O., Scandiuzzi, P. P., Valero, P. and Alrø, H. (2007). Learning Mathematics in a Borderland Position: Students' Foregrounds and Intentionality in a Brazilian Favela. *Working Papers on Learning* (7). Departments of Education, Learning and Philosophy. Aalborg University.

(3) Meaning

- The meaning of a classroom activity is first of all constructed by the students.
- This construction depends on the students' situation and, in particular, on the foreground of the students.
- Meaning construction depends on what the students may see as their possibilities.

Hope and meaning

- The construction of meaning gets it energy from the students' foreground. Meaning reflects motives, perspectives, hopes and aspirations
- However, meaning construction can be obstructed. A ruined foreground is a principal learning obstruction.
- Students' achievements (or lack of achievements) reflects their foregrounds.

(4) Mathematics in action

Ways of looking at mathematics:

- *The modern conception of mathematics:* Mathematics ensures a sublime way of obtaining understanding of nature. Mathematics is an indispensable resource for technological progress. Mathematic represents pure rationality.
- Maybe the modern conception of mathematics represents *myths* about mathematics? How to move beyond the modern conception of mathematics. *A critical conception on mathematics*?

The D'Ambrosio Paradox

"In the last 100 years, we have seen enormous advances in our knowledge of nature and in the development of new technologies...

And yet, this same century has shown us a despicable human behaviour. Unprecedented means of mass destruction, of insecurity, new terrible diseases, unjustified famine … are matched only by an irreversible destruction of the environment."

The Paradox continued

"Much of this paradox has to do with an absence of reflections and considerations of values in academics, particularly in the scientific disciplines, both in research and in education.

Most of the means to achieve these wonders and also these horrors of science and technology have to do with advances in mathematics" (page 443)

D'Ambrosio, U. (1994). Cultural Framing of Mathematics Teaching and Learning. In R. Biehler et al. (Eds.), *Didactics of Mathematics as a Scientific Discipline* (443-455). Dordrecht: Kluwer.

Mathematics in action

- As any other language, so also mathematics exercises a symbolic power.
- Mathematics is a powerful language of technology (understood broadly as including schemes of production, management, decision making, control, …)
- Mathematics makes part of a range of technological actions.
- Mathematics is an ingredient of our techno-nature.

Mathematics in action

- Mathematics can create a space of hypothetical situations (exercise a technological fantasy).
- Mathematics can support hypothetical reasoning.
- Mathematics can produce reasons for certain actions (justification, legitimation).
- Mathematics can be integrated in procedures for decision making, management, technological design, etc.
- Mathematics might dissolve responsibility.

Horrors and wonders

- No action, nor any action based on mathematics, can be counted as intrinsic “good”.
- Mathematics in action provides “horrors” and “wonders” in an unpredictable mixture.
- This is the basic observation of a critical conception of mathematics (contrary to the modern conception of mathematics).
- Skovsmose, O (2005). *Travelling Through Education*. Rotterdam: Sense Publishers.

(5) Uncertainty and doubt

- Much philosophy have been in search for certainty and solid epistemic foundations.
- I see the existence of such foundations as being a myth. Foundations constitute an utopia (in the literal sense of u-topia: no-place).
- For me doubt and uncertainty make part of the human condition.
- Skovsmose, O. (2009). *In Doubt: About Language, Mathematics, Knowledge and Life-Worlds*. Rotterdam: Sense Publishers.

(6) Critique

- The notion of critique has been related to notions like: analytical insight; well-specified political position; elaborated educational strategy.
- I suggest to develop the notion of critique without alliances with certainty. Instead I see critique as an expression of uncertainty and doubt.
- And as an expression of concerns as well. Concerns about ….

Appendix 1:
Terrible small numbers

Topic: Salmonella

Students: 15-16 years old

The teachers: Michael Skånstrøm

Henning Bødtkjer

See Alrø and Skovsmose (2002): *Dialogue and Learning in Mathematics Eduation: Intention, Refelction, Critique*. Dordrecht: Kluwer.

The aim

The aim of the project “Terrible small numbers” was to let the students explore questions related to statistics and probability.

In particular to explore the notions of reliability and responsibility with respect to mathematics in action.

Eggs

- One set of activities has to do with salmonella infected eggs.
- The whole population of eggs was brought into the classroom on a trolley from a supermarket.

The eggs

Sample of 'eggs' (Photo: Mikael Skånstrøm)

Samples

(50 eggs with sammonella out of 500. Samples of 10)

1	0	3	1	1	1
2	0	2	2	1	1
3	1	1	1	1	3
4	0	0	1	0	4
5	3	2	2	3	2

An issue of reliability

Some calculations

Infected eggs	Number of Samp	Freq	Cal freq
0	5	20%	34.4%
1	10	40%	39.1%
2	5	20%	19.5%
3	4	16%	5.6%
4	1	4%	1.0%

$$P(n) = K(50, n)\, K(450, 10-n) / K(500, n)$$

Greek or Spanish eggs?

Prices pr. Egg: 0.50 DKr

Salmonella control: 10 DKR per egg

Price pr sold egg: 1 DKr

Make a plan for decision making.

Make suggestion for advertising.

Some advertising

Sallemonella free eggs.

Tested for salmonella.

Free range chicken from Madrid.

An issue of responsibility

More Advertising

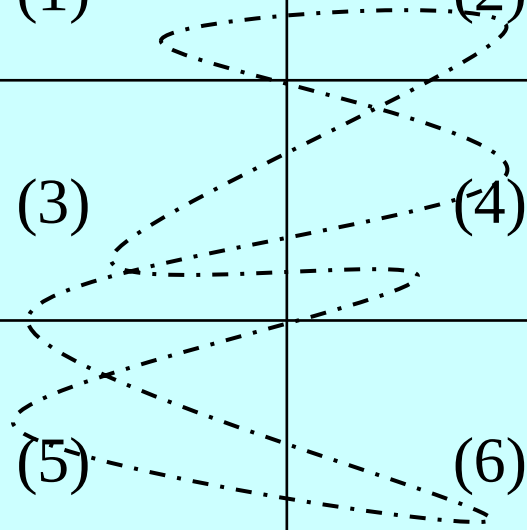
Eat only 9 of of 10.

Appendix 2: Milieus of learning

	Paradigm of exercises	Paradigm of investigations
References to mathematics	1	2
References to invented situations	3	4
References to features of real-life situations	5	6

Movement

	Paradigm of exercises	Paradigm of investigations
References to mathematics	(1)	(2)
References to invented situations	(3)	(4)
References to features of real-life situations	(5)	(6)



Comfort zone and risk zone

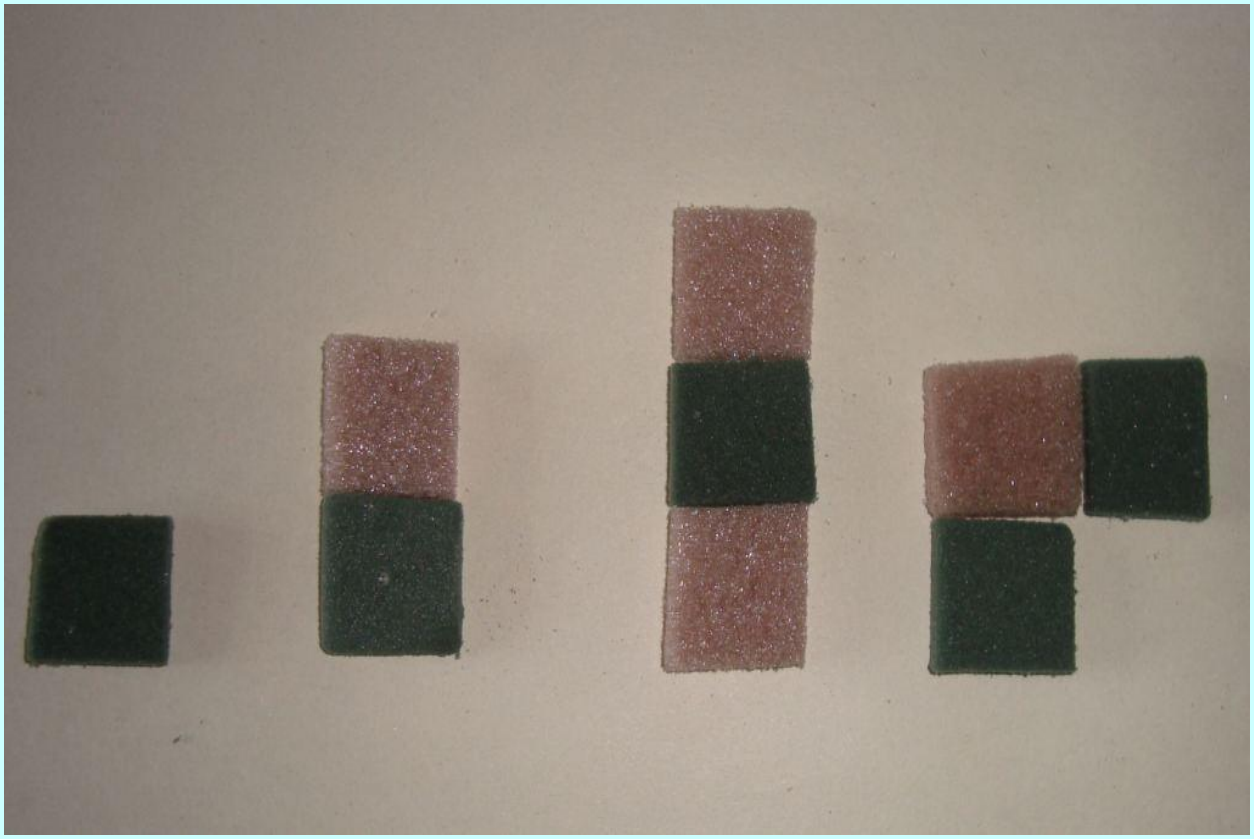
	Paradigma do exercício	Cenário para investigação
Referências à matemática pura	1	2
Referências à semi-realidade	3	4
Referências à realidade	5	6

Denival Biotto Filho (2008). *O desenvolvimento da matemacia no trabalho com projetos*. Unpublished Master Thesis. Universidade Estadual Paulista (UNESP), Instituto de Geociências e Ciências Exatas, Campus Rio Claro.

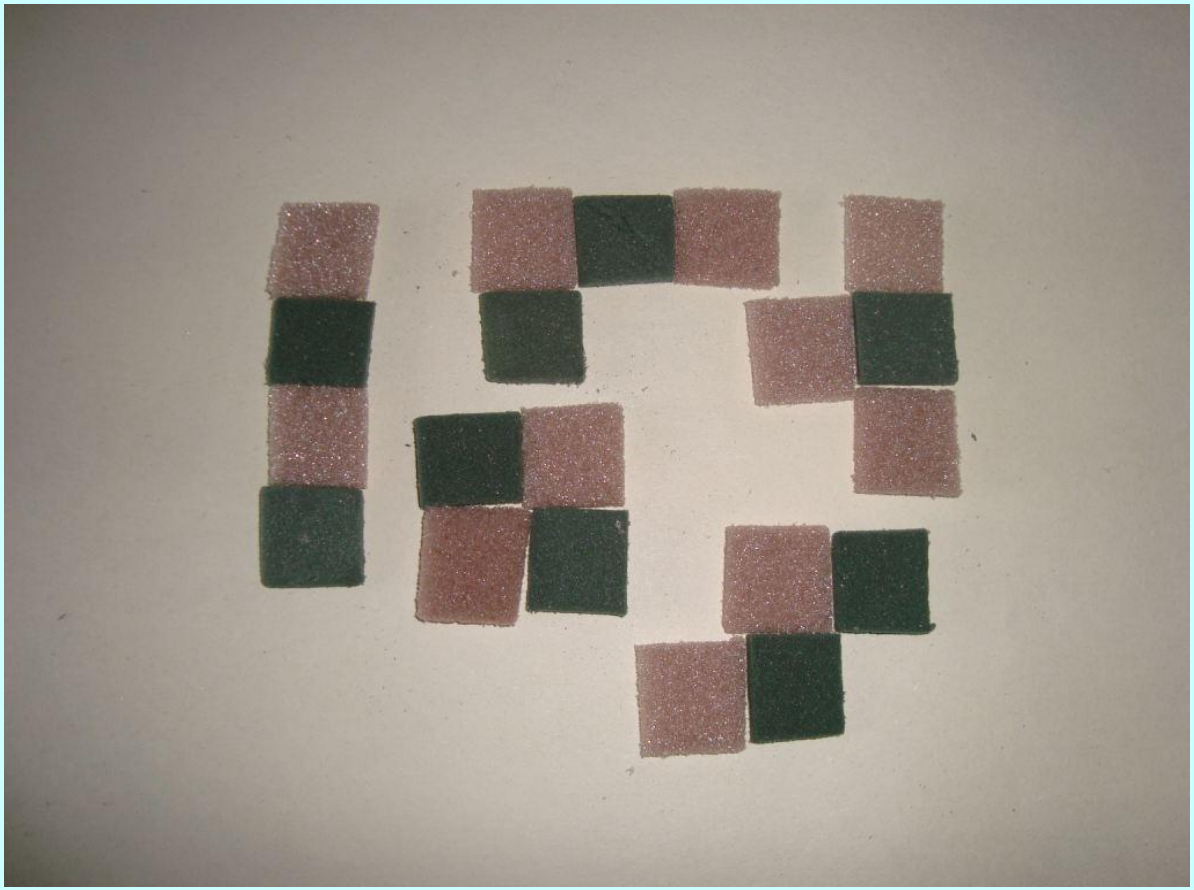
Appendix 3:

The animal farm

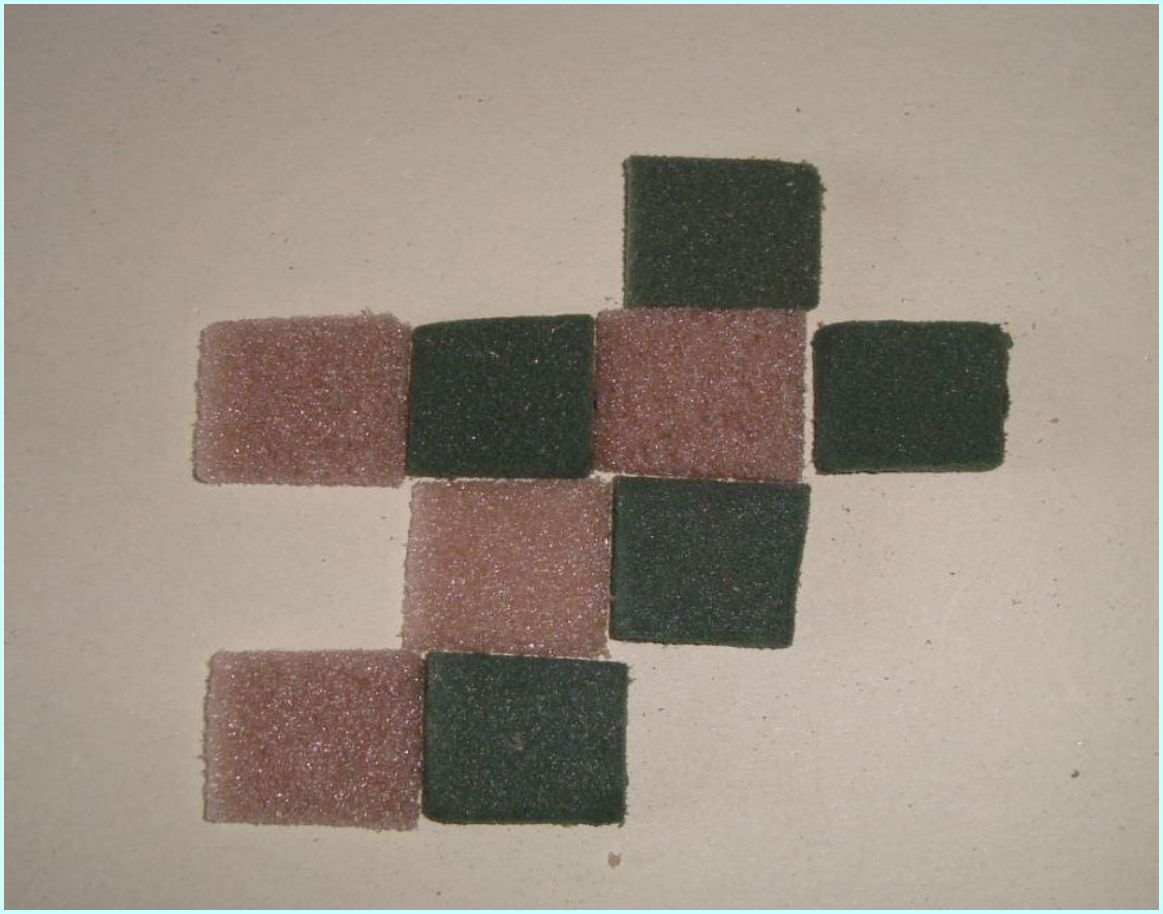
2-dimensional animals of size 1, 2 and 3)



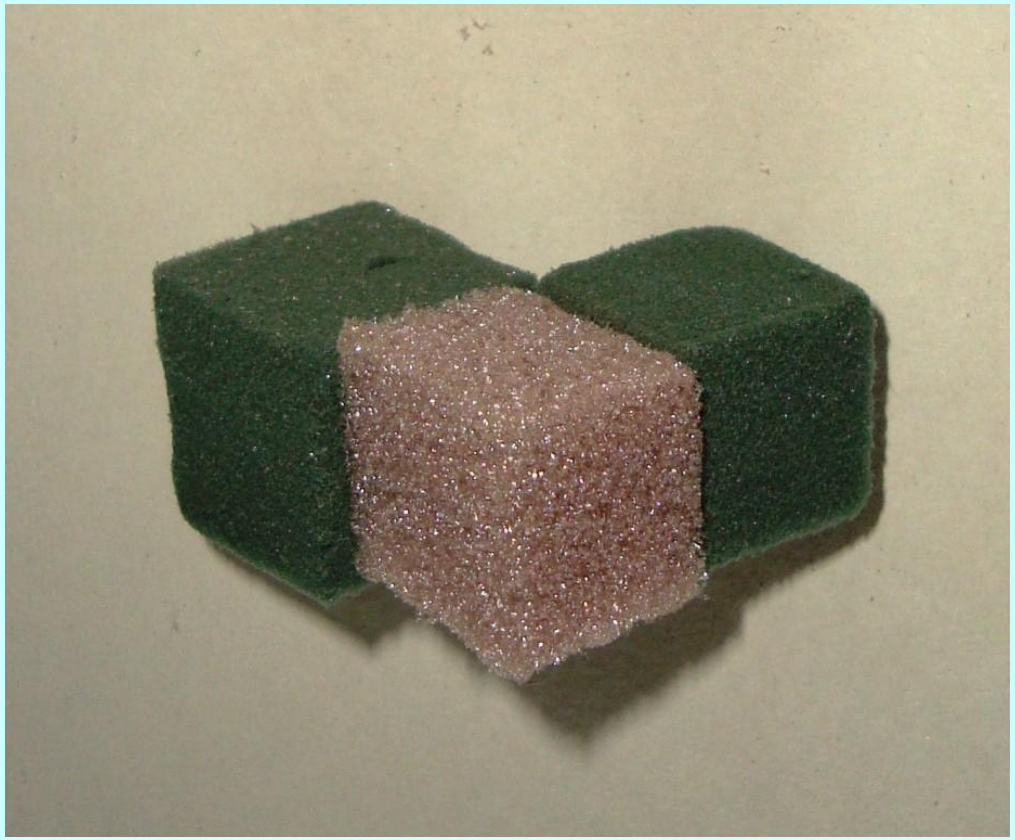
2-dimensional animals of size 4



A 2-dimensional animals of size 9



A 3-dimensional animals of size 3



3-dimensional animals of size 4

3-dimensional animals of size 5

Much more to explore

	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6
1-dimensional animals	1	1	1	1	1	1
2-dimensional animals	1	1	2	4		
3-dimensional animals	1	1	2			
4-dimensional animals	1					
5-dimensional animals						

The animals in their natural environment