# Thinking Algebraically

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- Algebra is an very important domain of school mathematics.
- A lot of researchers have extensively studied
  - the nature of algebra
  - the difference between arithmetic and algebra
  - the difficulties which students face while learning algebra
  - the reasons why students find algebra difficult

# Children's difficulties in algebra

1. Inadequate understanding of '=' sign
   - $4 + 5 = 9$
   - $3 + 4 = 2 + 5$
   - $7 + 4 - 6 + 9 = 11 = 11 - 6 = 5 = 5 + 9 = 14$
2. Difficulties with letters
   - $3a + 5 = 8$
   - $2a + 5b = 7\ ab$
3. Difficulties with notations and conventions
   - $a + 3 = 3a$ $\qquad$ $a \times 3 = 3a$
   - $(a + 3) \times 2 = a + 3 \times 2$

# Problems in equality sign

What to you think most children's response to this question will be?

$8 + 4 = \square + 5$

7? 12? 17?

The table below contains responses children from 30 primary classes in the US.

| Grade | 7 | 12 | 17 | 12 and 17 |
|---|---|---|---|---|
| 1 and 2 | **5** | 58 | 13 | 8 |
| 3 and 4 | **9** | 49 | 25 | 10 |
| 5 and 6 | **2** | 76 | 21 | 2 |

# Student's reasons for the wrong answer

Given below are responses of two students to the question,
*What number would you put in the box to make the number sentence true?*
$8 + 4 = \square + 5$

**Student 1:** [After a brief period] 12
**Teacher:** How do you know that its 12?
**Student 1:** Because thats the answer, 8 and 4 are 12. See, I counted, 8 [pause] 9, 10, 11, 12. See its 12.
**Teacher:** What about this 5 over here? [Pointing to the 5 in the number sentence]
**Student 1:** That's just there.

**Student 2:** Its 17.
**Teacher:** How do you know it is 17?
**Student 2:** Because I know that 8 and 4 is 12 and 5 more is 17.
**Teacher:** Why did you add all those numbers?
**Student 2:** Because it says to add.
**Teacher:** Okay. But these two numbers are over here on this side of the equal sign [points at $8 + 4$] and the 5 is over here.
**Student 2:** Yeah, but you have to add all the numbers. That's what it

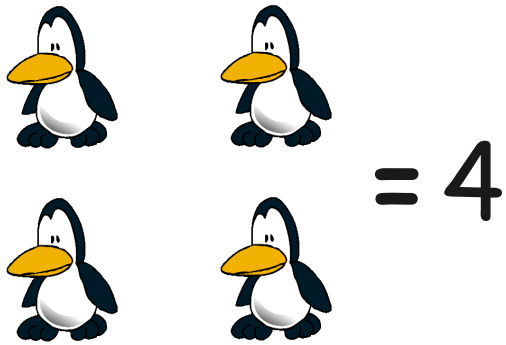
# Misconceptions about the equality sign

- They do not understand that equality sign denotes **relation** between two equal quantities
- They interpret the equal sign as a **command** to carry out a calculation
- For example:
  When a teacher asked her student if $2 = 1 + 1$ was true, the student said,
  '*I am not sure, it's backward, the one plus one is on the other side*'

# How can this be avoided?

A lot of times, we use the equal sign as a shorthand for a variety of reasons.

- Listing numerical characteristic of people or things:
  Rakesh $= 7$ years, Salma $= 30$ kg
- Designating the number of objects in a collection:




- Using equality to represent a string of calculations:

$$20 + 30 + 7 + 8$$

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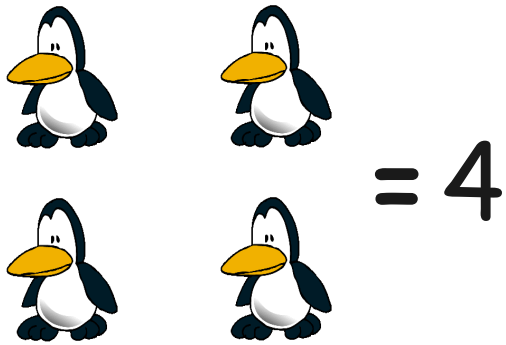



- Using equality to represent a string of calculations:

$$\begin{aligned} 20 + 30 + 7 + 8 &= 20 + 30 \\ &= 50 + 7 \\ &= 57 + 8 \\ &= 65 \end{aligned}$$

# What is the relationship between algebra and arithmetic?

- Algebra can be related to arithmetic in two ways:

1. Recognizing properties of operations like commutativity, associativity or distributivity.
   - For example : $a + b = b + a$
2. Solutions of numerical equations
   - For example : $x + y = 12$

# Nature of solution: Arithmetic or Algebraic?

Given below are responses of two students to the question,
*What number would you put in the box to make the number sentence true?*
$8 + 4 = \square + 5$

**Student 1:** [After some time] It's 7.
**Teacher:** How do you know that it is 7?
**Student 1:** Well, $8 + 4$ is 12. So I had to figure out what to go with 5 to make 12, and I figured out that it was 7.
**Teacher:** So why did you want to figure out what to put with 5 to make 12?
**Student 1:** Because I had 12 over here [pointing to the left of the equal to sign], so I had to have 12 over here [pointing to the right of the equal to sign]. And 5 and 7 is 12.

**Student 2:** [*Very quickly*] Seven
**Teacher:** How do you know it is 7?
**Student 2:** Well, I saw the 5 over here[ pointing to the 5 in the number sentence], was one more than 4 over here [pointing out], so the number in the box had to be one less than 8. So its 7.
**Teacher:** Thats very interesting. Let's try another one. How about
$57 + 86 = \square + 84$?
**Student 2:** [Almost immediately] Thats easy. 59
**Teacher:** That was quick! **Student 2:** It's just like the other one. Its just two more because 84 is two less.

# Comparing the two answers

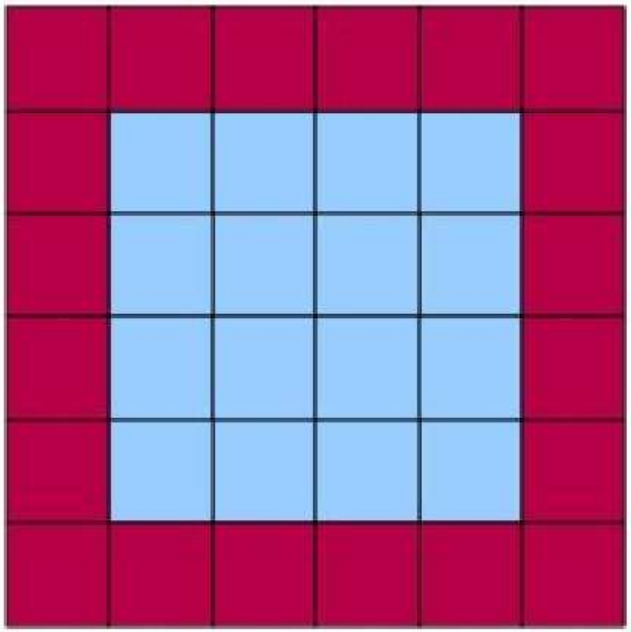
Though both the solutions given by the students were correct there was a big difference in the strategies used.
What was the difference between the solutions given by the two students?

- Student 1 solved the problem **arithmetically**.
  - He added $8 + 4$ to get 12 and then subtracted 5 from 12 to get 7.
  - When he comes across a problem like: $57 + 86 = \square + 84$ or similar problems with bigger numbers he will calculate the answer from problem to problem.
- Student 2 solved the problem **algebraically**.
  - She compared the two sides and found a relation between the two pairs of addends.
  - Hence she could $57 + 86 = \square + 84$ very quickly.

# How can you help the student think '*algebraically*'?

- Work out 'true or false' number sentences.
  - Number sentences can help them make conjectures.
- Use patterns to develop relational thinking.
  - Number sentences with a pattern helps the students read the pattern and find relations themselves.
- Questions which make them generalize

# Questions which might help



How many different ways can you find to count the border tiles of an $4 \times 4$ pool without counting them one at a time?
Use the expression you get to predict how many border tiles will be needed if the pool is $75 \times 75$.

These are some of the answers had students had got

- $4 \times 5$
- $(4 \times 4) + 4$
- $(4 \times 6)$ - 4
- $6^2 - 4^2$

# Aspects of *algebraic thinking*

- Reading patterns
- Generalizations
- Making conjectures
- **Proving them**

*THANK YOU*