Approaches to Learning Fractions

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Learning difficulties of Fractions in the School curriculum

- Unfamiliar way of parsing fraction
- A complex concept which is a combination of several sub-constructs
- Composition is multiplicative in nature in contrast to whole numbers

The sub-construct theory of fractions

- Measure interpretation
- Share interpretation
- Fraction as an operator
- Ratios
- Fraction as part of a whole.

The Measure interpretation of fractions

In the measure sub-construct, fraction is considered as a number, which conveys the quantitative personality of fractions, how big the fraction is. A unit fraction is defined (i.e., 1/a) and used repeatedly to determine a distance (Lamon, 2001; Marshall, 1993)

The share/quotient interpretation of Fractions

Within the quotient sub-construct, any fraction can be seen as the result of a division situation. In particular, the fraction *x*/*y* indicates the *numerical value* obtained when *x* is divided by *y*, where *x* and *y* represent whole numbers (Kieren, 1993).

Ratios using fraction notation

The ratio sub-construct of fractions conveys the notion of a comparison between two quantities; therefore, it is considered as a comparative index, rather than a number (Carraher, 1996).

Fractions as operators

In the operator interpretation of fractions, rational numbers are regarded as functions applied to some number, object, or set (Behr et al., 1993; Marshall, 1993).

Fraction as a part of a whole

The part-whole sub-construct of fractions is defined as a situation in which a continuous quantity or a set of discrete objects are partitioned into parts of equal size (Lamon, 1999; Marshall, 1993).

From this perspective, the numerator of the fraction must be less than or equal to the denominator.

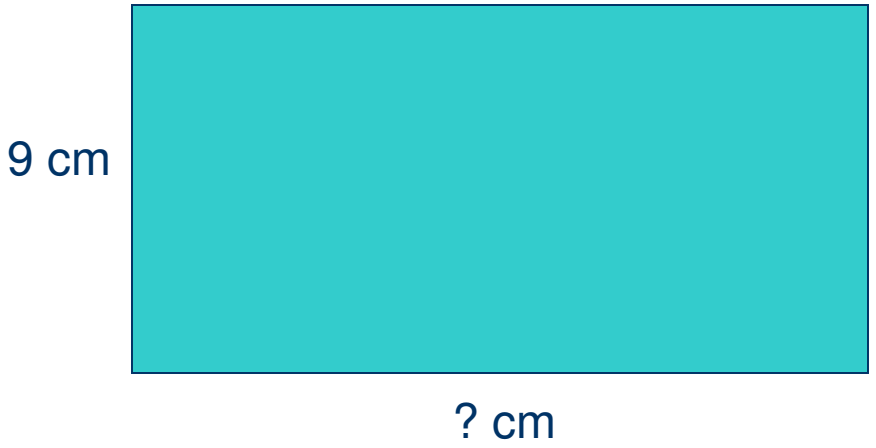
Fractions in our textbooks

- Language used in the textbook (part-part)
- Definitions are procedure oriented
  - When numerator is less than the denominator the fraction is called as proper fraction.
- A paragraph from our textbook
  - The shortest way to find the equivalent fraction in the simplest form is to find HCF of the numerator and denominator, then divide both of them by HCF and get the equivalent fraction.

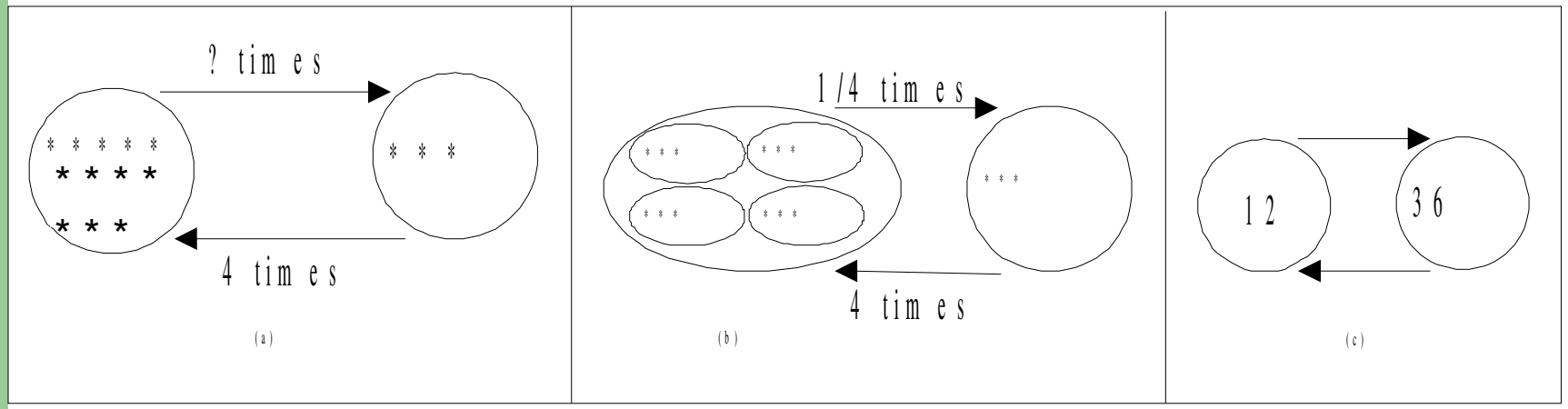
Work at HBCSE

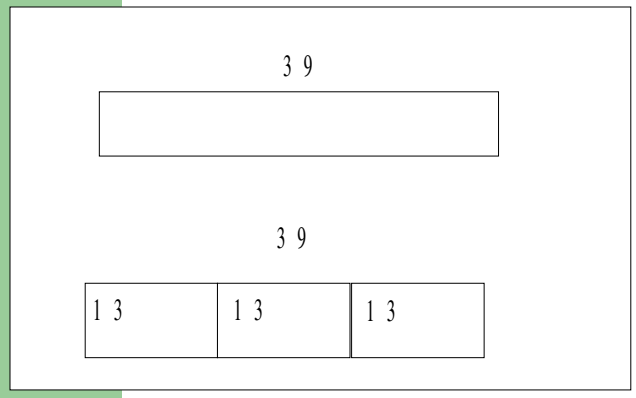
- Students' difficulties in understanding multiplicative relationships
- Understanding fractions through integration of share and measure approach
- Representation of equivalent fractions by making groups in which everybody gets the same share
- Students' representing problems involving fractions on their own/with peers

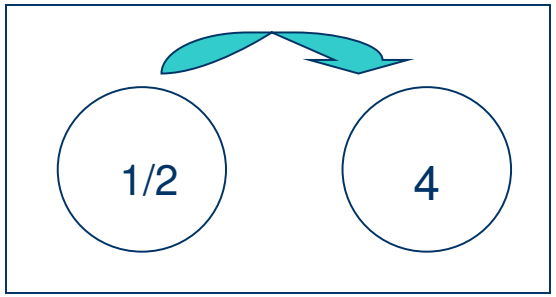
Multiplicative relations

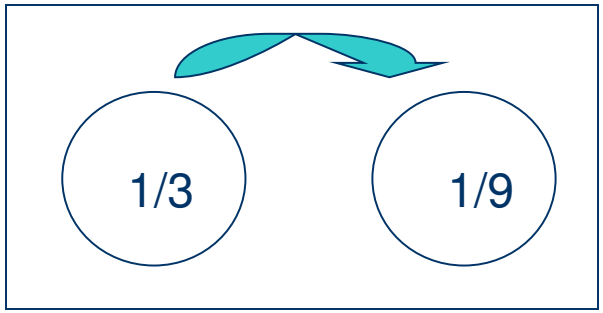


Multiplicative relations



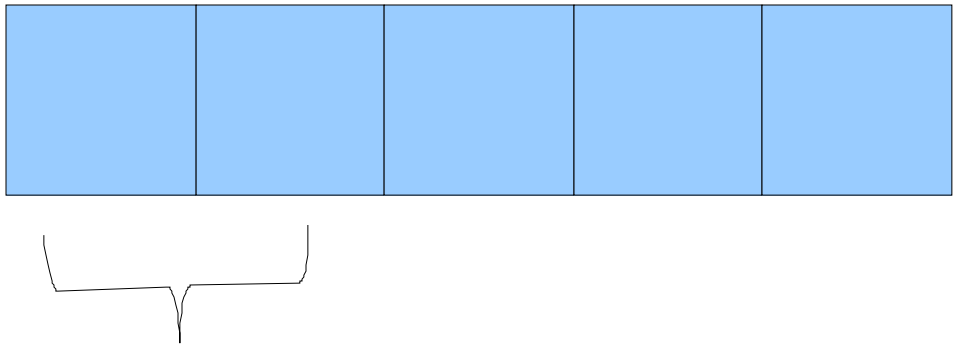




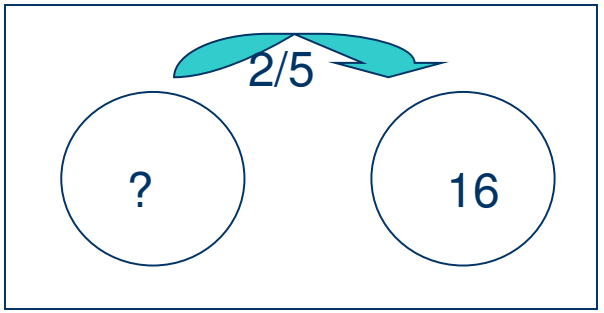


Multiplicative relations

Following representation was made by a student when it was asked to him to solve _____ x 2/5 = 16. The reasoning that the student provided was like this *if the whole is divided among five parts and if we take 2 parts of it we are getting 16, so we need to find the whole.*



16



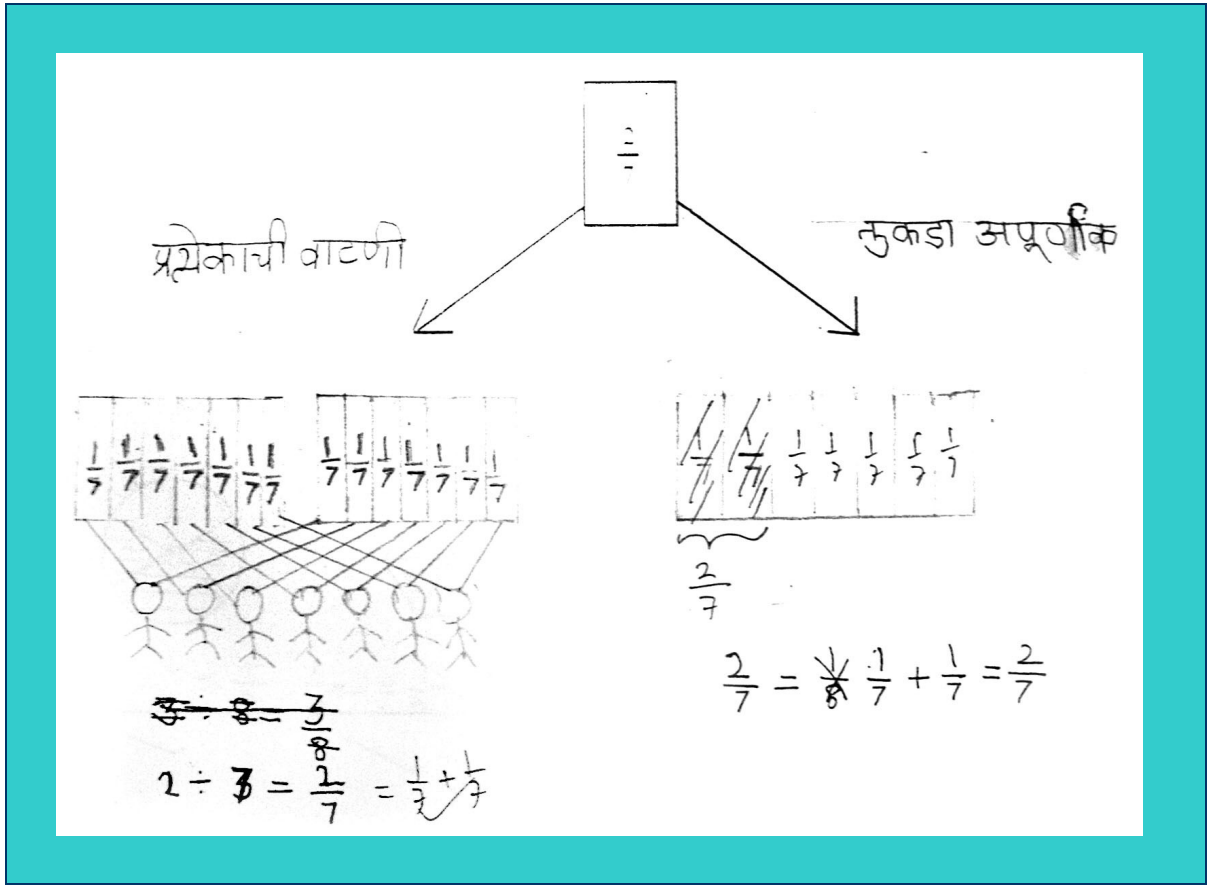
Understanding fractions differently

Integration of measure and share approach - both these meanings complement each other.

The share meaning explains the process denoted by the fraction symbol.

The idea of unit fraction helps students to decide how much is the share of each child after completion of the process.

Integrating Share and measure approach



Teaching approaches for share interpretation

- Understanding fair sharing through situations
- Finding shares in terms of fraction
  - From simple fractions to general fractions
- Context used for sharing –
  - Chapatis (on board using Navnirmiti material)
  - Rectangular cakes
  - Sharing chai

Teaching approaches for share interpretation

- In sharing chapatis making different equal parts (e.g. 1/10) is difficult. But reference to a whole as a chapati is easier for students.
- Rectangular whole allows many ways of equi-partitioning.
- Sharing tea problems became more interesting, as we worked with three sets of values – total number of cups of tea, number of people to share and money per person.

Sharing problem : Chai

Cost of chai is Rs. 6. If 3 persons are sharing 2 chai then the students found that the share of each child is 2/3 chai and each person will have to pay Rs. 4 .

Teaching approaches for Measure interpretation

- Measuring things –
  - A coloured paper strip was used by students to measure length of doors, windows and tables accurately
- Students arrived at some mixed fractions as their answers
- All there answers were in terms of halves and quarters

Informal units in measurement

- Various informal units re-discovered in students' measurement –
  - Half of half which they understood as a quarter
  - Half of a quarter / quarter of a half
  - Quarter of a quarter

Understanding of equi-partitioning

- A paper with some part shaded was given to the students to find out how much is the shaded part?
- One of the students group claimed that sometimes parts (e.g ½ , ¼) which looked unequal can be equal if their weight is same. They did weighing in the classroom, to find this.

Students' reasoning about fractions

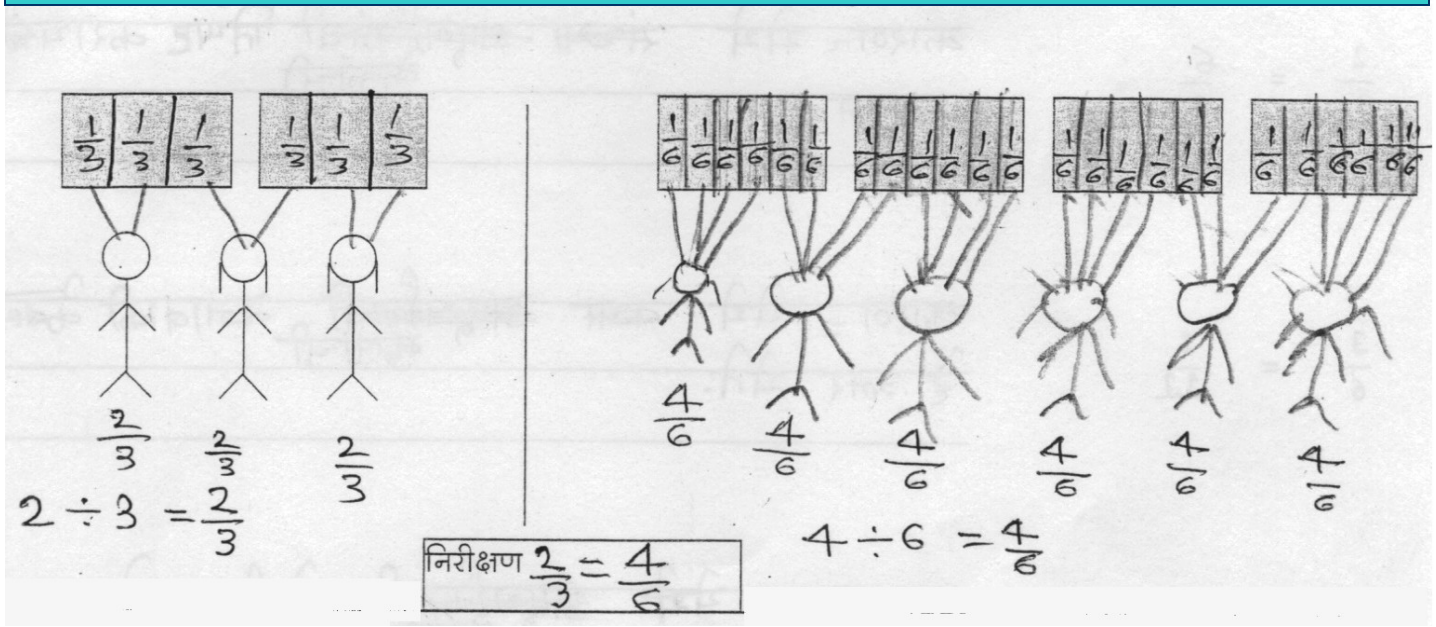
In comparison tasks students drew on both the measure and the share interpretations.

- Fractions with the same numerator:
  - As the number of cakes to share are same, the group where more number of children are there will have a smaller share.
  - As the number of pieces are same what matters is the size of the unit.
- Comparing fractions with half:
  - Students reasoned that the fraction is equal to half when the number of cakes is exactly half the number of children or when the number of pieces taken is exactly half the total number of pieces.

Students' reasoning about fractions

- In comparing the fractions 4/5 and 6/7,
  - *Even though both the fractions 4/5 and 6/7 need one more piece to complete a whole. 4/5 needs one piece of 1/5 and 6/7 needs one piece of 1/7. but 1/5 is more than 1/7 as one cake is shared among 5 children only, hence 4/5 is away from the whole.*
- Improper fractions were understood as
  - *as number of cakes are more than the number of children to share. Obviously each child will get at least one cake*

Equivalent situations

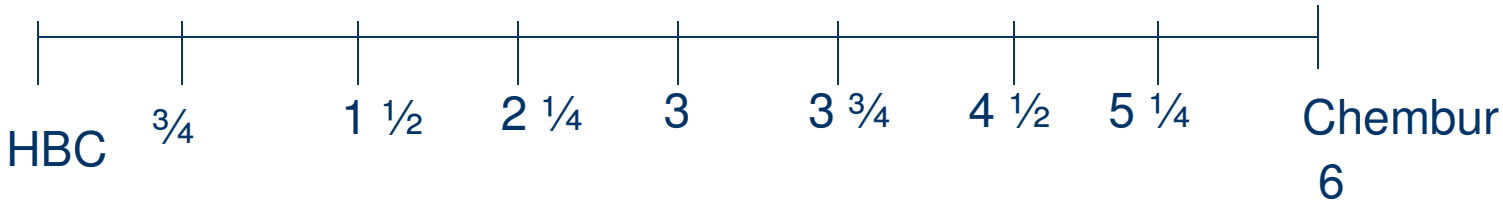


Equivalent fractions

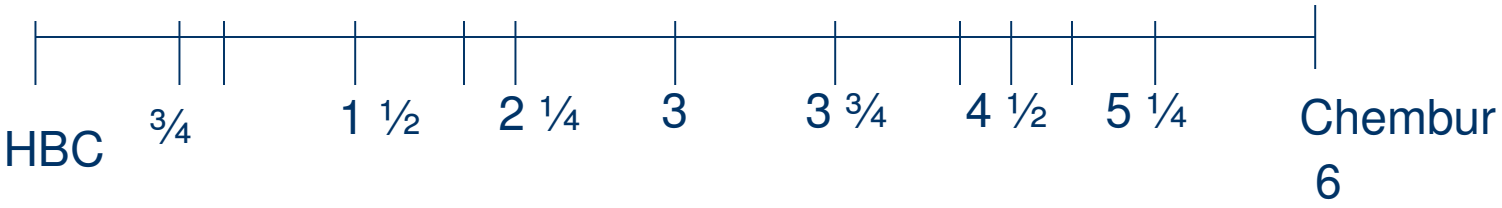
- If the situations are equivalent then the share of each child is in each of the situations is same.
- Children started arguing that 3/5 is same as 9/15 as we can make three groups of 3/5 in it , with every group getting the same share.

Students' representation in Warm-up

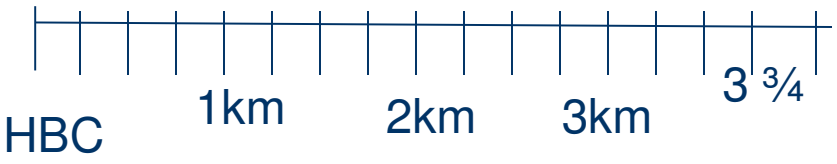
BMC decided to tar road from HBCSE to Chembur. The distance between these two is 6 km. If tarring machine completes ¾ km road everyday, how many days it will take to finish the tarring work?



Representations…

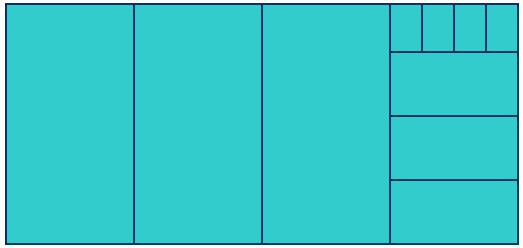


For modified problem when asked that tarring machine does ½ km per day and total distance is 3 ¾ km.



Approximation for 1/3

- Share following among three children



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